# Nuclear Reactor Physics Notes

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The collision density $N(x)$ in the case of zero absorption is

$$N(x) = q \int_{x-1}^{x} \alpha^{x-y} \; N(y)\, dy, \quad \text{where } h(y) = 1 \tag{1}$$

With non-negligible absorption

$$N(x) = q \int_{x-1}^{x} \alpha^{x-y} h(y) \; N(y)\, dy \; \text{ where } h(y) = \frac{\Sigma_s(y)}{\Sigma_s(y) + \Sigma_a(y)} \tag{2}$$

As seen in previous lessons

$$N(x) = [\Sigma_s(x) + \Sigma_a(x)]\, \Phi(x) \tag{3}$$

It follows

$$\begin{aligned} h(x)[\Sigma_s(x) + \Sigma_a(x)] = \Sigma_s(x) \Longrightarrow h(x) N(x) &= \\ &= h(x)[\Sigma_s(x) + \Sigma_a(x)]\,\Phi(x) \\ &= \Sigma_s(x)\,\Phi(x) = R(x) \end{aligned} \tag{4}$$

$$R(x) = qh(x) \int_{x-1}^{x} \alpha^{x-y} h(y) \; N(y)\, dy = qh(x) \int_{x-1}^{x} \alpha^{x-y} R(y)\, dy \tag{5}$$

Let's say $y = x + \eta$. So that

$$R(x) = qh(x) \int_{-1}^{0} \alpha^{-\eta} R(x+\eta)\, d\eta \Longrightarrow R(x+\eta) = R(x) + \eta R'(x) + \eta^2 \frac{R''(x)}{2!} \tag{6}$$

By replacing

$$R(x) = qh(x) \int_{-1}^{0} \alpha^{-\eta} R(x+\eta)\, d\eta + qh(x) \int_{-1}^{0} \alpha^{-\eta} \eta R'(x)\, d\eta + qh(x) \int_{-1}^{0} \alpha^{-\eta} \eta^2 \frac{R''(x)}{2} d\eta \tag{7}$$

Place

$$M_0 = \int_{-1}^{0} \alpha^{-\eta} d\eta, \quad M_1 = \int_{-1}^{0} \alpha^{-\eta} \eta d\eta, \quad M_2 = \int_{-1}^{0} \alpha^{-\eta} \eta^2 d\eta \tag{8}$$

we will write

$$R(x) = qh(x)\left[M_0 R(x) + M_1 R'(x) + M_2(x) R''(x)\right] \tag{9}$$

Calculating the integrals (8):

$$M_0 = \frac{1-\alpha}{\ln \frac{1}{\alpha}}, \quad M_1 = \frac{a \ln \frac{1}{\alpha} - (1-\alpha)}{\left(\ln \frac{1}{\alpha}\right)^2}, \quad M_2 = \frac{2(1-\alpha) - \alpha\left[\left(\ln \frac{1}{\alpha}\right)^2 + 2 \ln \frac{1}{\alpha}\right]}{\left(\left(\ln \frac{1}{\alpha}\right)^2\right)^3} \tag{10}$$

It immediately occurs that

$$qM_0 = 1, \quad qM_1 = -\frac{\xi}{\ln \frac{1}{\alpha}} \tag{11}$$

Let's imagine for now it is $R''(x) = 0$. Then:

$$R(x) = h(x)\left[R(x) - \frac{\xi}{\ln\frac{1}{\alpha}}R'(x)\right] \Longrightarrow \frac{R'(x)}{R(x)} = -\frac{\xi}{\ln\frac{1}{\alpha}} \cdot \frac{1-h(x)}{h(x)} \tag{12}$$

Integrating

$$\ln\frac{R(x)}{R(0)} = -\frac{\ln\frac{1}{\alpha}}{\xi}\int_0^x \frac{1-h(y)}{h(y)}dy \Longrightarrow \frac{R(x)}{R(0)} = \exp\left[-\frac{\ln\frac{1}{\alpha}}{\xi}\int_0^x \frac{1-h(y)}{h(y)}dy\right] = \frac{\Phi(x)}{\Phi(0)} \tag{13}$$

Moving on from relative lethargy $x = \frac{\ln(E_0/E)}{\ln(1/\alpha)}$ to the energy:

$$\frac{\Phi(E)}{\Phi(E_0)} = \exp\left[-\frac{1}{\xi}\int_{E_0}^E \frac{1-h(E')}{h(E')}dE'\right] = \exp\left[-\frac{1}{\xi}\int_{E_0}^E \frac{\Sigma_a(E')}{\Sigma_s(E')}dE'\right] \tag{14}$$

Now let's imagine it is $R''(x) \neq 0$. By the (12), deriving both members

$$R'(x) = h'(x)R(x) + h(x)R(x) - \frac{\xi}{\ln\frac{1}{\alpha}}R''(x)h(x) - \frac{\xi}{\ln\frac{1}{\alpha}}R'(x)h'(x) \tag{15}$$

Being $\Sigma_a \sim 1$ it can be assumed that

$$h(x) = \frac{\Sigma_s(x)}{\Sigma_s(x) + \Sigma_a(x)} \sim \text{constant}$$

Therefore $h'(x) = 0$ and we will write

$$R''(x) = R'(x)\frac{1-h(x)}{qM_1 h(x)} \Longrightarrow R''(x) = f(R'(x)) \tag{16}$$

We substitute in the (9) $R''(x) = f(R'(x))$

$$R(x) = h(x)R(x) - \frac{\xi}{\ln(1/\alpha)}h(x)R'(x) + \frac{M_2}{2}R'(x)\frac{1-h(x)}{M_1} \tag{17}$$

Let's say

$$\frac{M_2}{2M_1} = -\frac{\gamma}{\ln(1/\alpha)}$$

where $\gamma = \gamma(\alpha)$ and replacing it is easy to get

$$R(x)\Sigma_a(x) = -\frac{R'(x)}{\ln(1/\alpha)}\left[\xi\Sigma_s(x) + \gamma\Sigma_a(x)\right] \tag{18}$$

By integrating we have

$$\frac{R(x)}{R(0)} = \exp\left[-\ln\frac{1}{\alpha}\int_0^x \frac{\Sigma_a(y)}{\xi\Sigma_s(y) + \gamma\Sigma_a(y)}dy\right] \tag{19}$$

Or

$$\frac{R(E)}{R(E_0)} = \exp\left[-\int_{E_0}^E \frac{\Sigma_a(E')}{\xi\Sigma_s(E') + \gamma\Sigma_a(E')}dE'\right] \tag{20}$$

Let's look at the case of hydrogen $H_1^1$. We remember that

$$\xi = 1 + \frac{(A-1)^2}{2A} \ln \frac{A-1}{A+1} \Longrightarrow \xi = \gamma_1 (\alpha) = 1 \tag{21}$$

The collision density per unit energy interval is:

$$F(E) = \frac{h(E_0)}{E_0 (1-\alpha)} \int_{E_0}^{E} h(E') F(E') \frac{dE'}{E'} \quad \text{where } \alpha E_0 < E < E_0 \tag{22}$$

With $\alpha = 0$ becomes valid the first expression of $F(E)$ because the range $0 < E < E_0$ is real; while the second makes no sense because $E < 0$. Noting that $\frac{h(E_0)}{E_0} = F(E_0)$ we have:

$$\begin{aligned} -F(E_0) + F(E) &= \int_{E_0}^{E} h(E') F(E') \frac{dE'}{E'} \\ \Longrightarrow dF(E) &= h(E) \frac{F(E)}{E} dE \\ \Longrightarrow \int_{E_0}^{E} \frac{dF(E')}{F(E')} &= -\int_{E_0}^{E} h(E') \frac{dE'}{E'} \\ \Longrightarrow \ln \frac{F(E)}{F(E_0)} &= -\int_{E_0}^{E} h(E') \frac{dE'}{E'} \\ \Longrightarrow F(E) &= \frac{h(E_0)}{E_0} \exp\left[-\int_{E_0}^{E} \frac{h(E')}{E'} dE'\right] \end{aligned} \tag{23}$$

Remembering the hypothesis $\Sigma_a(E) \sim 1$ it can be assumed that $h(E) \sim 1$. It follows

$$F(E) \sim \frac{1}{E} \tag{24}$$