Nuclear binding energy

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1 Nuclear binding energy

Rest mass of some nuclear subparticles:

- Neutron $n = 1.008982$ amu
- Proton $p = 1.007593$ amu
- Electron (or β^- particle) $e = 0.00054876$ amu
- Positron (or β^+ particle) $e^+ = 0.00054876$ amu
- Neutrino $\nu \simeq 0$
- Meson π_+ or $\pi_- = 0.14994$ amu
- Meson μ_+ or $\mu_- = 0.11351$ amu
- Meson $\pi_0 = 0.14509$ amu
- α particle=$\mathrm{He}_2^4 = 4.003873$ amu

In a nuclear reaction we must check the following mass and charge balances:

$$\mathrm{H}_{Z_1}^{A_1} + \mathrm{K}_{Z_2}^{A_2} \text{ reactants } = \mathrm{L}_{Z_3}^{A_3} + \mathrm{M}_{Z_4}^{A_4} \quad \text{products} \tag{1}$$

$$\mathrm{M}_{A_1} + \mathrm{M}_{A_2} = \mathrm{M}_{A_3} + \mathrm{M}_{A_4} + \text{Energy} \qquad \text{exothermic reaction} \tag{2}$$

$$\mathrm{M}_{A_1} + \mathrm{M}_{A_2} = \mathrm{M}_{A_3} + \mathrm{M}_{A_4} - \text{Energy} \qquad \text{endothermic reaction} \tag{3}$$

$$Z_1 + Z_2 = Z_3 + Z_4 \tag{4}$$

The subject of nuclear stability will be not treated at this point; a semi-empirical approach, based on nuclear masses, will be used here.

1.1 Mass Defect and Binding Energy

Definition 1 **Nuclear Binding Energy**= *the work required to disintegrate a nucleus completely into Z protons and (A-Z) neutrons.*

The binding energy may be calculated from the equations:

$$\text{Mass defect} = \Delta_{md} = Zm_H + (A - Z)\, m_n - M \tag{5}$$

where:
M = experimental mass of the nuclide including the masses of the atomic electrons
m_H = mass of the hydrogen atom m_n = mass of the neutron

Since the masses are given in amu the corresponding energy is

$$1 \text{ amu} \cdot c^2 = 1.67 \cdot 10^{-24} \cdot \left(2.997930 \cdot 10^{10}\right) = 1.492 \cdot 10^{-3} \,\text{erg} \simeq 931 \,\text{MeV} \tag{6}$$

So that the binding energy is:

$$B_E = \Delta_{md} c^2 = 931 \Delta_{md} \text{ in MeV} \tag{7}$$

The average binding energy per nucleon of a nuclide is:

$$f_B = \frac{B_E}{A} \tag{8}$$

which is called the **binding fraction**.

It is known that for mass number higher than 80 the function f_B vs A is a line. This means that the total binding energy B_E increase roughly as the square of the mass number. The mean binding fraction f_B in the elements of low A is low, but over a considerable range it is close to 8 MeV. Therefore the total B_E is approximately proportional to the mass number A. Except for the elements of lowest mass number the radius of a nucleus can be expressed by:

$$R = 1.5 \cdot 10^{-13} \cdot A^{1/3} \tag{9}$$

The volume of a nucleus is directly proportional to A. This means that all atomic nuclei containing the same constituents, namely, neutrons and protons, have essentially the same density. Under this point of view one can conclude that a nucleus behaves like a liquid with short-range forces operating between the constituent particles.

Example 2 *Calculate the nuclear binding energy of* $C^{12.010}$.

M_H =mass of $H^1 = 1.00814$ amu; M_n =mass of neutron= 1.00898 amu
M =mass of nuclide $C^{12.010} = 12.010$ amu
Mass defect= $\Delta_{md} = Z M_H + (A - Z)\, M_n - M = 6 \cdot 1.00814 + (12 - 6) \cdot 1.00898 - 12.010 =$
$= 0.09272$ amu corresponding to $B_E = 0.09272 \cdot 931 \,\text{MeV} = 86.32 \,\text{MeV}$.

1.2 Semi-empirical calculation of B_E

$$B_E\,(\text{MeV}) = 14.0\, A - 13.0\, A^{2/3} - 19.3 \frac{(A - 2Z)^2}{A} - 0.585 \frac{Z^2}{A^{1/3}} + \delta_{spin} \tag{10}$$

Z protons	$N = A - Z$ neutrons	δ_{spin}
even	even	$33 A^{3/4}$
odd	odd	$-33 A^{3/4}$
even	odd	0
odd	even	0

	A	Z	N= A–Z	δ (even Z even N)	δ (odd Z odd N)	δ (others)	B_E calculated MeV	B_E calculated MeV	B_E calculated MeV	f_B	
Xe	140	54	86	0,81081	other	0	1141	-	-	8,15	
Cs	140	55	85	other	-0,81081	0	-	1144	-	8,17	
Ba	140	56	84	0,81081	other	0	1149	-	-	8,21	
La	147	57	90	other	other	0	-	-	1193	8,11	
Ce	140	58	82	0,81081	other	0	1152	-	-	8,23	
Sr	94	38	56	1,09312	other	0	796	-	-	8,47	
Y	94	39	55	other	-1,09312	0	-	798	-	8,49	
Zr	94	40	54	1,09312	other	0	802	-	-	8,53	
Kr	97	36	61	other	other	0	-	-	794	8,19	
Xe	133	54	79	other	other	0	-	-	1098	8,26	
Ca	40	20	20	2,07477	other	0	342	-	-	8,54	
U	238	92	146	0,544604	other	0	1798	-	-	7,55	

1.3 Magic and Semi-Magic Numbers

A nucleus excited emits electromagnetic radiation (gamma rays) of discrete energies just as an atom does. It has discrete quantum or levels of energy. There are certain numbers of nucleons that make a nucleus nuclearly stable. The most stable nuclei found in nature have a number of Z protons or $(A - Z)$ neutrons equal to one of the following numbers:

$$2, \quad 8, \quad 20, \quad 50, \quad 82, \quad 126$$

these are called **magic numbers**. For example: H_2^4, O_8^{16}, Ce_{58}^{140}, Pb_{82}^{208}

Other nuclei showing good stability but less than those above have Z or $(A - Z)$ equal to:

$$14, \quad 28, \quad 40$$

these are called **semu–magic numbers**.