

## Rango di una matrice. Minori estratti. Plotting di matrici

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Per quanto detto in una sezione precedente, *Mathematica* gestisce i vettori e matrici attraverso una lista. Precisamente, un vettore è una lista. Ad esempio, se vogliamo definire il vettore  $v = (1, -1, 4)$  dello spazio vettoriale  $\mathbb{R}^3$ , scriviamo:

```
In[1]:= v = {1, -1, 4};
```

Una matrice è, invece, una lista di liste. Ad esempio, si consideri la matrice  $4 \times 3$

```
In[2]:= A = {{0, 2, 1}, {1, -1, 0}, {-1, 1, 0}, {1, 1, 1}};
```

```
In[3]:= A // MatrixForm
```

```
Out[3]//MatrixForm=
```

$$\begin{pmatrix} 0 & 2 & 1 \\ 1 & -1 & 0 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

```
In[4]:= A // Dimensions
```

```
Out[4]= {4, 3}
```

Il rango della matrice  $A$  si calcola con l'istruzione **MatrixRank**

```
In[5]:= A // MatrixRank
```

```
Out[5]= 2
```

Volendo determinare i minori estratti di ordine 3:

```
In[6]:= Minors[A, 3] // MatrixForm
```

```
Out[6]//MatrixForm=
```

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

I minori di ordine 2:

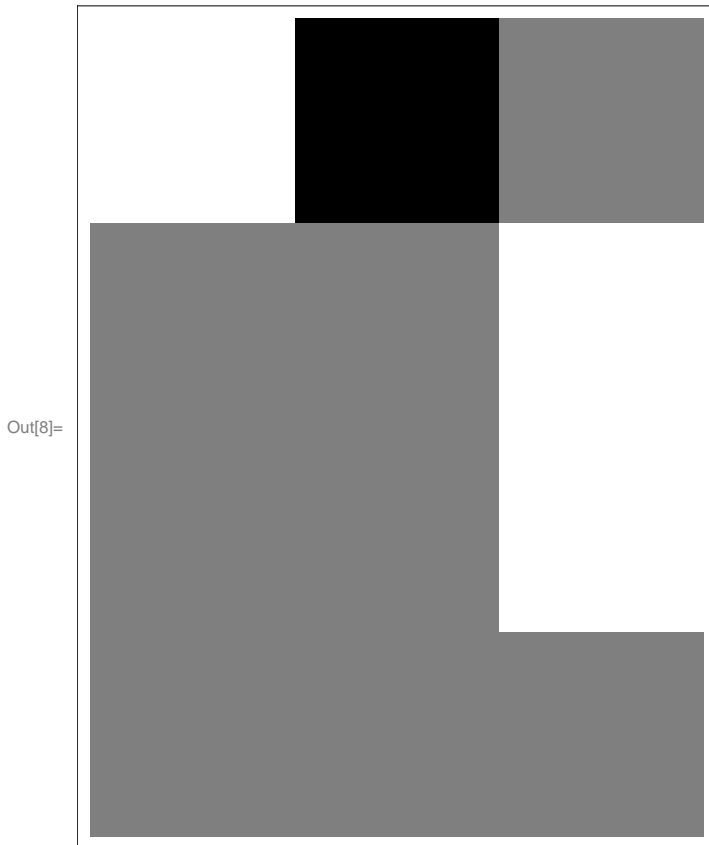
```
In[7]:= Minors[A, 2] // MatrixForm
```

```
Out[7]//MatrixForm=
```

$$\begin{pmatrix} -2 & -1 & 1 \\ 2 & 1 & -1 \\ -2 & -1 & 1 \\ 0 & 0 & 0 \\ 2 & 1 & -1 \\ -2 & -1 & 1 \end{pmatrix}$$

Una matrice può essere "plottata" con l'istruzione **ArrayPlot**

```
In[8]:= A // ArrayPlot
```



Consideriamo quest'altra matrice generata con l'istruzione `RandomInteger`

```
In[9]:= matrix = RandomInteger[1, {50, 50}]
```

```
Out[9]= {{1, 1, 0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 1, 1, 1, 1, 1, 1, 0, 1, 1,
  0, 0, 0, 1, 0, 0, 0, 1, 1, 1, 0, 0, 0, 1, 0, 1, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0},
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```

```

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```

```
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```

```
In[10]:= matrix // ArrayPlot
```

```
Out[10]=
```

