

## Calcolo di limiti di funzioni razionali fratte

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```
In[1]:= (*ver. 6.0*)
```

```
In[2]:= Limit[
  f[x],
  x → x0
]
```

```
Out[2]= Limit[f[x], x → x0]
```

```
In[3]:= Limit[
  (*espressione analitica
  della funzione*)
  
$$\frac{x^2 - 2x + 1}{x - 1},$$

  (*punto di accumulazione a cui
  tende la variabile x*)
  x → 1
]
```

```
Out[3]= 0
```

```
In[4]:= lim[f_, x0_, k_] :=
  Limit[
    (*espressione analitica
    della funzione*)
    f[x],
    (*punto di accumulazione a cui
    tende la variabile x*)
    x → x0,
    (*limite destro: k=-1;
    limite sinistro: k=+1 *)
    Direction → k
  ]
```

Se non siamo interessati ai limiti destro ( $x \rightarrow x_0^+$ ) e sinistro ( $x \rightarrow x_0^-$ ), inseriamo  $k=1$ , ripetendo poi il calcolo per  $k=-1$ .

```
In[5]:= Clear[f]
```

```
In[6]:= f[x_] := 
$$\frac{x^2 - 3x + 2}{x^2 + x - 6}$$

```

```
In[7]:= lim[f, 2, 1]
```

```
Out[7]= 
$$\frac{1}{5}$$

```

```
In[8]:= lim[f, 2, -1]
```

```
Out[8]= 
$$\frac{1}{5}$$

```

Calcolando il limite con carta e penna, vediamo che il rapporto si presenta nella forma indeterminata 0/0. Per rimuovere l'indeterminazione dobbiamo scomporre in fattori i polinomi a numeratore e denominatore. Possiamo controllare la correttezza dei nostri calcoli, nel seguente modo:

```
In[9]:= g[x_] = f[x] // Factor
```

```
Out[9]= 
$$\frac{-1 + x}{3 + x}$$

```

```
In[10]:= lim[g, 2, 1]
```

```
Out[10]= 
$$\frac{1}{5}$$

```

Per avere un'idea più precisa del comportamento della funzione in un intorno di  $x_0$  tracciamo un grafico "locale":

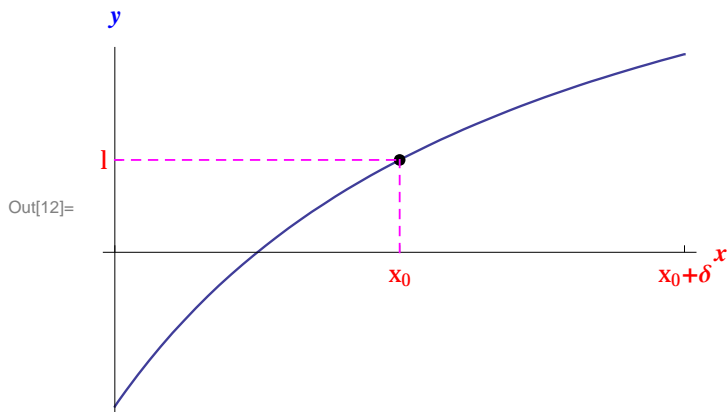
```
In[11]:= local[f_, x0_, δ_] := Block[
  (*dichiaro le variabili locali*)
  {
    xmin,
    xmax,
    plot
  },
  (*esprimo le variabili locali
  in termini delle variabili di input*)
  xmin = x0 - δ;
  xmax = x0 + δ;
  plot = Plot[
    (*scrivo la funzione*)
    f[x],
    (*traccio il grafico in un intorno di x0*)
    {x, xmin, xmax},
    PlotRange → Automatic,
    AxesLabel → {
      Style[x, Medium, Bold, Red],
      Style[y, Medium, Bold, Blue]
    },
    LabelStyle → Directive[Red, Bold]
  ,
  PlotStyle → {
    Thickness[0.004]
  },
  Ticks → {
    {
      {x0, "x0"}, {x0 + δ, "x0+δ"}, {x0 - δ, "x0-δ"}
    },
    {
      {lim[f, 2, 1], "l"}
    }
  },
  TicksStyle → Directive[
    FontFamily → "Times New Roman",
    FontSize → 12
  ],
  Epilog → {
    {
      PointSize[0.02], Point[{x0, lim[f, 2, 1]}]
    },
    {
      Dashed,
      Hue[5 / 6],
      Thickness[0.003],
```

```

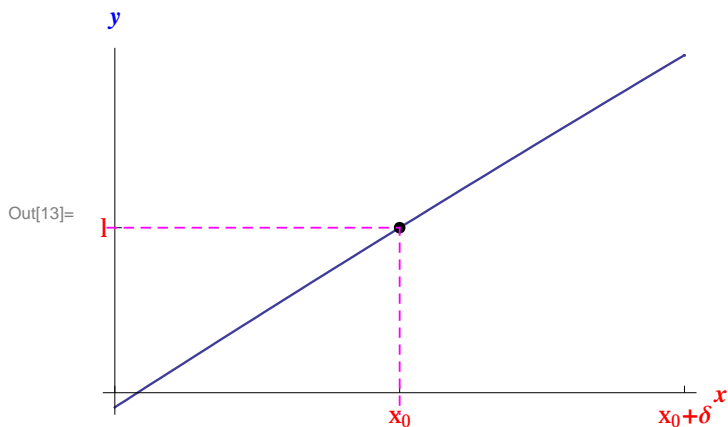
Line[{{x0, 0}, {x0, lim[f, 2, 1]}}]
},
{
Dashed,
Hue[5 / 6],
Thickness[0.003],
Line[{{0, lim[f, 2, 1]}, {x0, lim[f, 2, 1]}}]
}
}
]
]

```

In[12]:= `local[f, 2, 2]`



In[13]:= `local[f, 2, 0.1]`

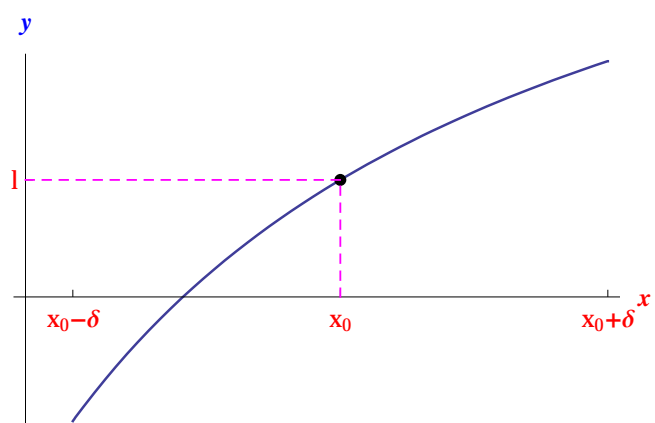
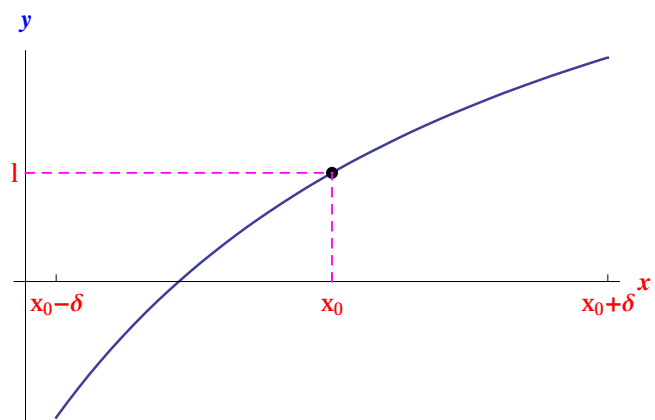
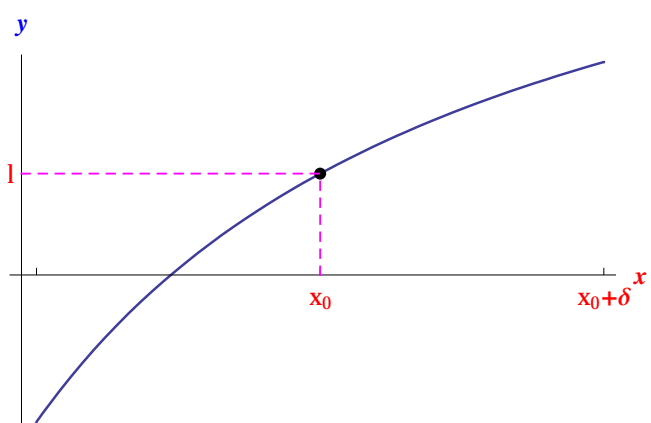
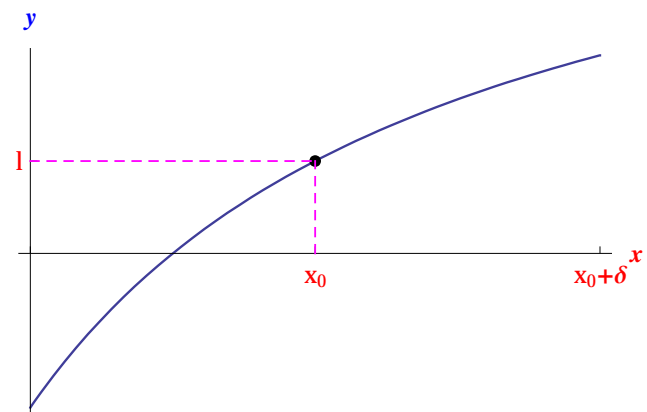


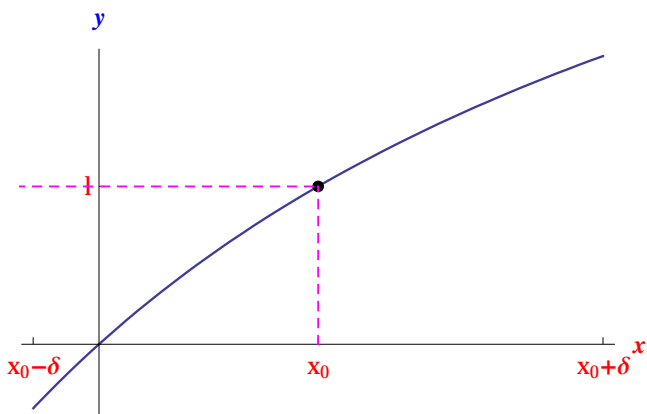
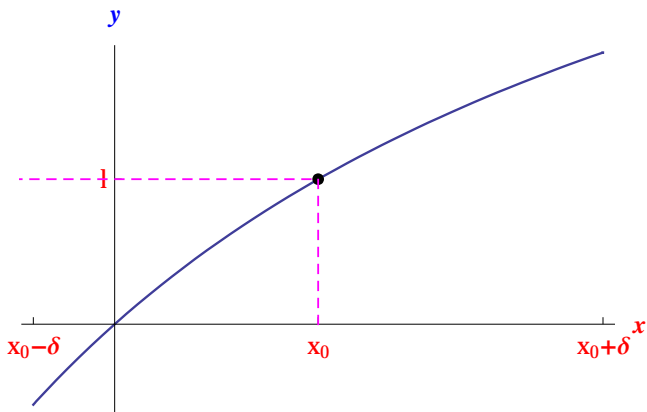
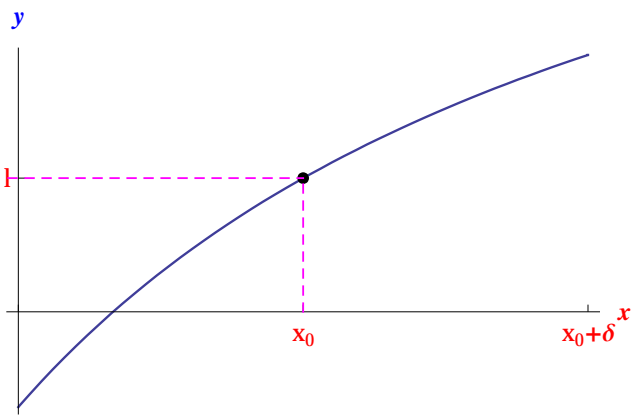
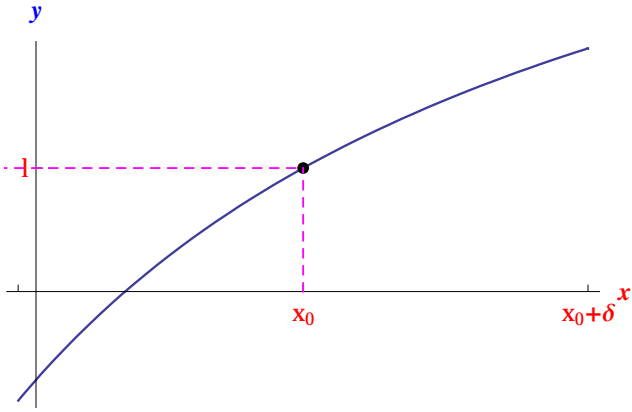
L'utilizzo di un ciclo `Do` ci permette di creare un'animazione grafica per valori decrescenti dell'ampiezza dell'intorno:

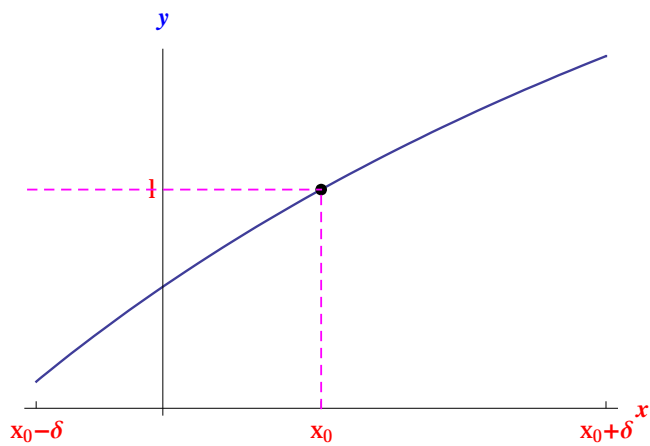
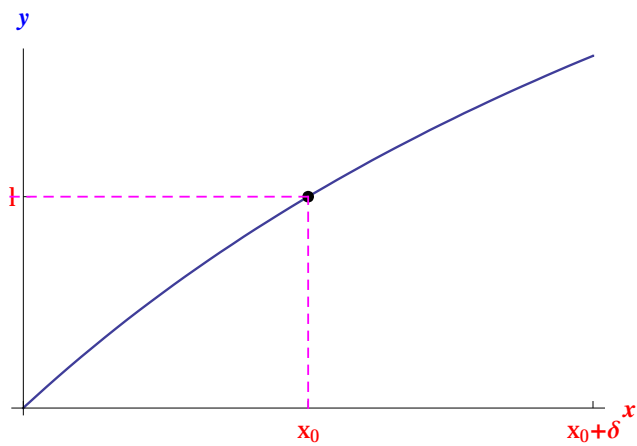
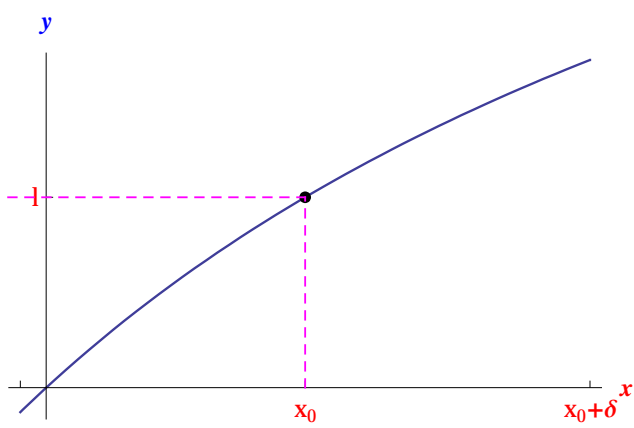
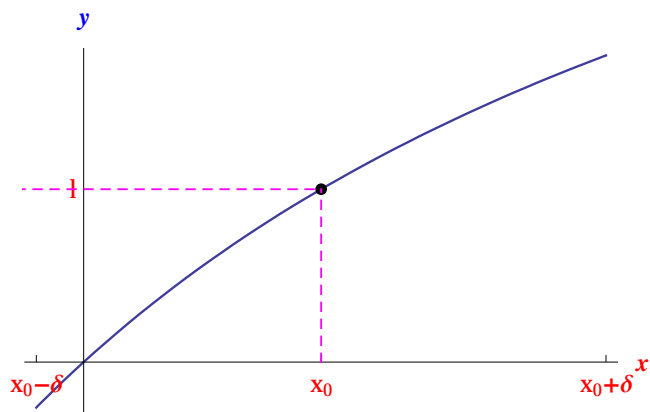
```

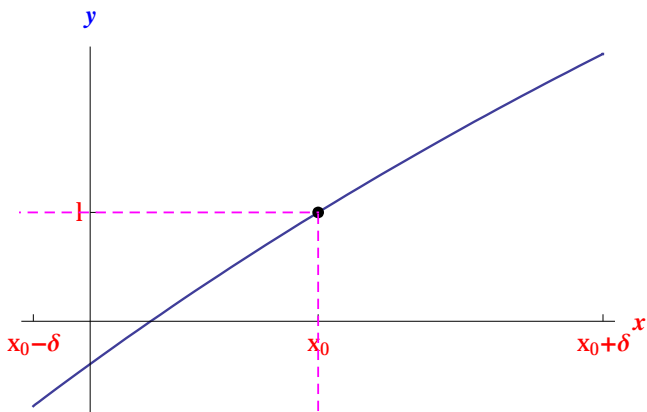
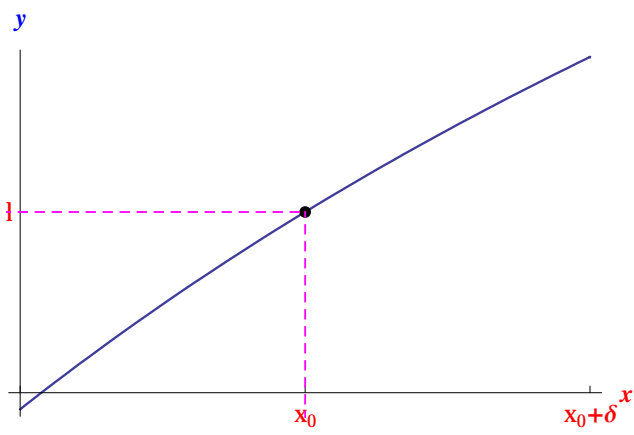
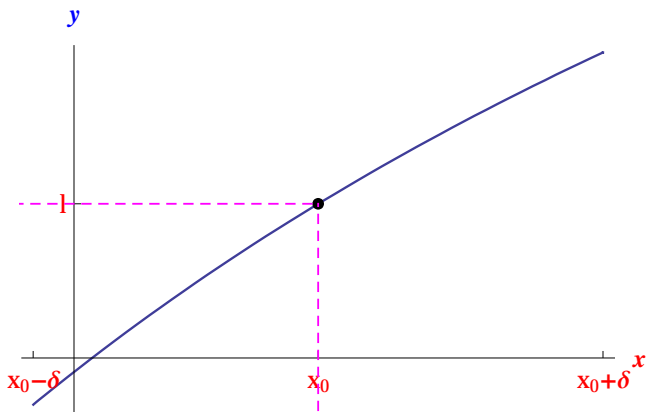
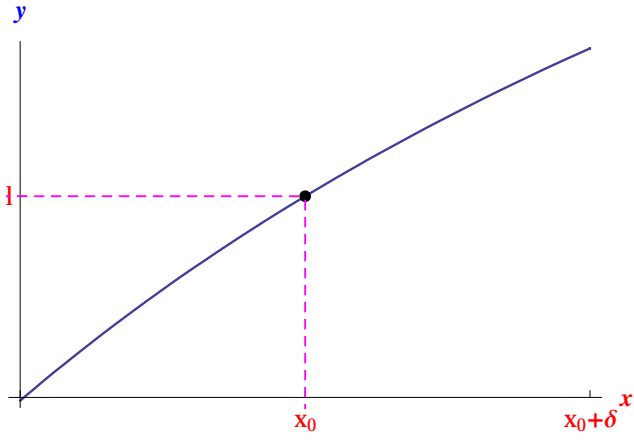
In[14]:= animazione = Do[
(*stampa a video il grafico
per assegnati valori di  $\delta$ *)
Print[
local[f, 2,  $\delta$ ],
(*iteratore*)
{ $\delta$ , 2, 0.2, -0.1}
];

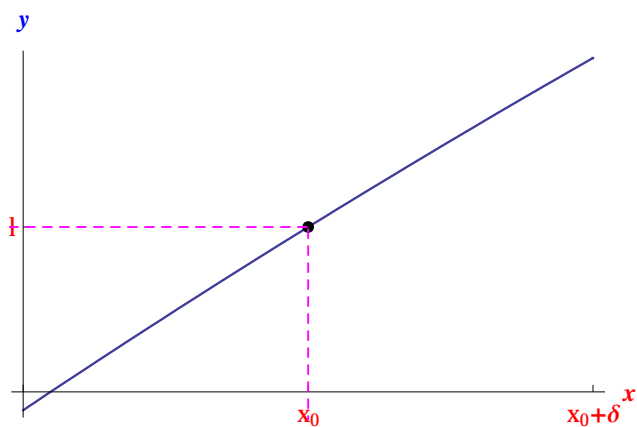
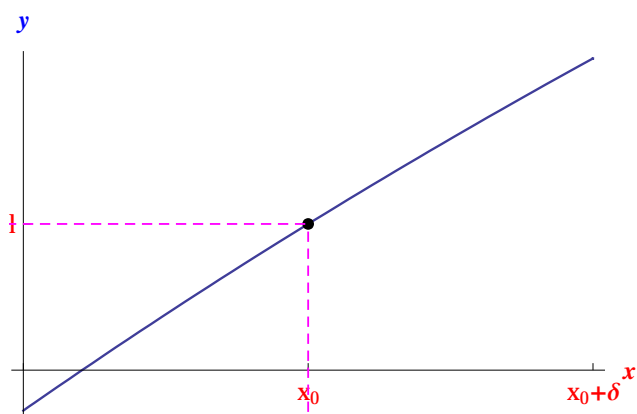
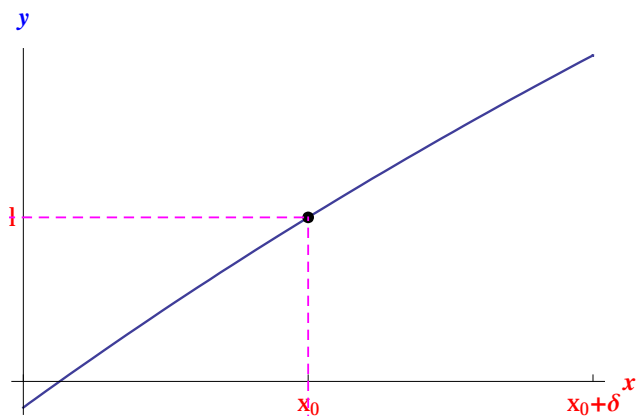
```











```
In[15]:= Clear[f]
```

```
In[16]:= f[x_] :=  $\frac{x^2 - x - 6}{x^3 + 5x^2 + 8x + 4}$ 
```

```
In[17]:= lim[f, -2, 1]
```

```
Out[17]=  $-\infty$ 
```

```
In[18]:= lim[f, -2, -1]
```

```
Out[18]=  $\infty$ 
```

```
In[19]:= Clear[local]
```

```
In[20]:= local[f_, x0_, delta_] := Block[
  (*dichiaro le variabili locali*)
  {
    xmin,
```



```

xmax,
plot
},
(*esprimo le variabili locali
 in termini delle variabili di input*)
xmin = x0 -  $\delta$ ;
xmax = x0 +  $\delta$ ;
plot = Plot[
  (*scrivo la funzione*)
  f[x],
  (*traccio il grafico in un intorno di  $x_0$ *)
  {x, xmin, xmax},
  PlotRange  $\rightarrow$  Automatic,
  AxesLabel  $\rightarrow$  {
    Style["x", Medium, Bold, Red],
    Style["y", Medium, Bold, Blue]
  },
  LabelStyle  $\rightarrow$  Directive[Red, Bold]
,
  PlotStyle  $\rightarrow$  {
    Thickness[0.004]
  },
  Ticks  $\rightarrow$  {
    {-2.05, -2, -1.95, -1.90},
    Automatic
  },
  TicksStyle  $\rightarrow$  Directive[
    FontFamily  $\rightarrow$  "Times New Roman",
    FontSize  $\rightarrow$  12
  ],
  Exclusions  $\rightarrow$ 
  {
    x = x0
  },
  Epilog  $\rightarrow$  {
    {
      Dashed,
      Hue[5/6],
      Thickness[0.003],
      Line[{{x0, -500}, {x0, 500}}]
    },
    {
      Hue[1],
      Thickness[0.003],
      Arrow[{{-1.94, 50}, {-1.999, 50}}]
    },
    {
      Hue[1],
      Thickness[0.003],
      Arrow[{{-1.994, 150}, {-1.994, 400}}]
    },
    Text[
      "x $\rightarrow$ -2",
      {-1.97, 100}
    ],
    Text[
      "+ $\infty$ ",

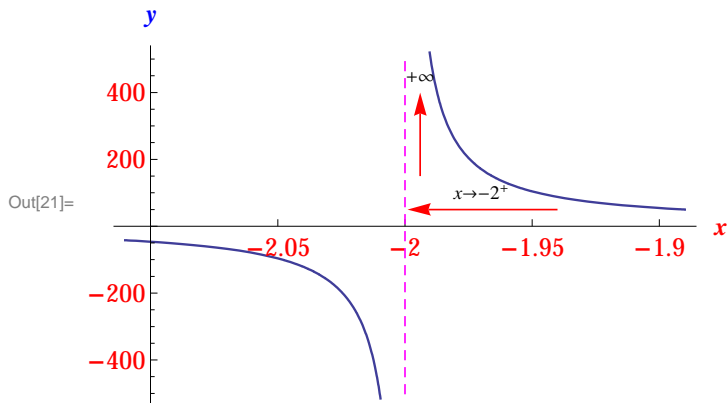
```

```

      {-1.994, 450}
    ]
  }
]
]

```

```
In[21]:= local[f, -2, -0.11]
```



```
In[22]:= Clear[
  (*cancello f dal kernel*)
  f,
  (*la variabili x va in conflitto con x0,
  quindi la rimuovo dal kernel*)
  x
]

```

```
In[23]:= f[x_] :=  $\frac{x^4 - 8 x^2 + 16}{x^3 - 8}$ 
```

```
In[24]:= lim[f, 2, -1]
```

Out[24]= 0

```
In[25]:= g[x_] =  $\frac{x^4 - 8 x^2 + 16}{x^3 - 8}$  // Factor
```

Out[25]=  $\frac{(-2 + x) (2 + x)^2}{4 + 2 x + x^2}$

```
In[26]:= lim[g, 2, -1]
```

Out[26]= 0

```
In[27]:= lim[g, 2, 1]
```

Out[27]= 0

```
In[28]:= Clear[local]
```

```

In[29]:= local[f_, x0_,  $\delta$ _] := Block[
  (*dichiaro le variabili locali*)
  {
    xmin,
    xmax,
    plot
  },
  (*esprimo le variabili locali
  in termini delle variabili di input*)
  xmin = x0 -  $\delta$ ;
  xmax = x0 +  $\delta$ ;
  plot = Plot[
    (*scrivo la funzione*)
    f[x],
    (*traccio il grafico in un intorno di x0*)
    {x, xmin, xmax},
    PlotRange → Automatic,
    AxesLabel → {
      Style[x, Medium, Bold, Red],
      Style[y, Medium, Bold, Blue]
    },
    LabelStyle → Directive[Red, Bold]
  ,
  PlotStyle → {
    Thickness[0.004]
  },
  Ticks → {
    {
      {x0, "x0"}, {x0 +  $\delta$ , "x0+ $\delta$ "}, {x0 -  $\delta$ , "x0- $\delta$ "}
    },
    {
      {lim[f, 2, 1], "l"}
    }
  },
  TicksStyle → Directive[
    FontFamily → "Times New Roman",
    FontSize → 12
  ],
  Epilog → {
    {
      PointSize[0.02], Point[{x0, lim[f, 2, 1]}]
    }
  }
]
]

```

In[30]:= `local[f, 2, 3]`

Out[30]=

