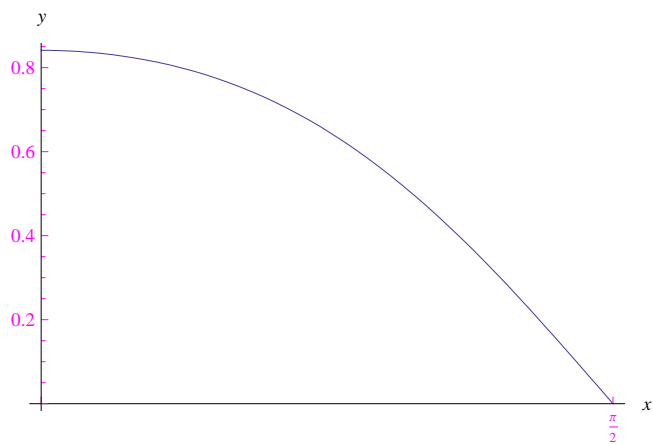


## Ricorsione di $f_\lambda(x)=\sin(\lambda\cos(x))$

Marcello Colozzo - <http://www.extrabyte.info>

```
In[10]:= ff[x_, λ_] := Sin[λ * Cos[x]]
```

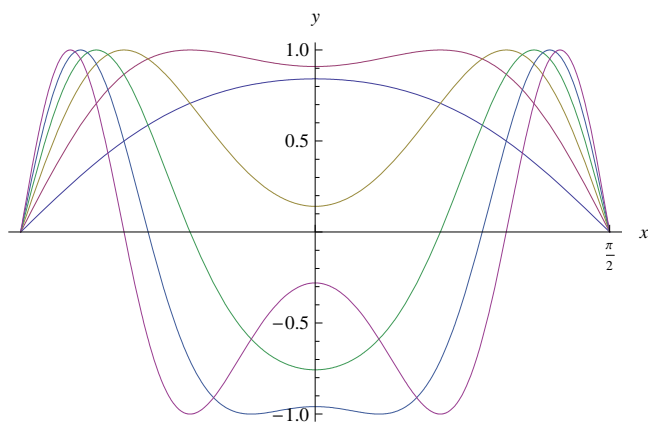
```
plot = Plot[  
  ff[x, 1],  
  {x, 0, π/2},  
  Ticks →  
  {  
    {0, π/2}  
  },  
  AxesLabel →  
  {  
    "x", "y"  
  },  
  TicksStyle → Directive[  
    Hue[5/6], 9]  
]  
]
```



```

plots = Plot[
  Evaluate[
    Table[
      ff[x, λ],
      {λ, 1, 6}
    ]
  ],
  {x, -π/2, π/2},
  Ticks →
  {
    Table[k, {k, 0, 2 π, π/2}]
  },
  AxesLabel →
  {
    "x", "y"
  }
]

```



```

Dff[x_, λ_] = D[
  ff[x, λ],
  x
]
-λ Cos[λ Cos[x]] Sin[x]

```

```
x0 = 1.4;
```

```
f[x_] := ff[x, 0.8]
```

```

iterazione[n_] := NestList[
  f,
  x0,
  n
]

```

Per  $n = 30$ :

```
iterazione[30]
```

```

{1.4, 0.135555, 0.712224, 0.569197, 0.624013, 0.604575, 0.61169, 0.609114,
 0.61005, 0.609711, 0.609834, 0.609789, 0.609805, 0.609799, 0.609802, 0.609801,
 0.609801, 0.609801, 0.609801, 0.609801, 0.609801, 0.609801, 0.609801,
 0.609801, 0.609801, 0.609801, 0.609801, 0.609801, 0.609801, 0.609801, 0.609801}

```

Le iterazioni sembrano convergere a 0.609801:

```
Clear[iterazione]
```



```

iterazione[10^2]
{0.609801, 0.609801, 0.609801, 0.609801, 0.609801, 0.609801,
 0.609801, 0.609801, 0.609801, 0.609801, 0.609801, 0.609801,
 0.609801, 0.609801, 0.609801, 0.609801, 0.609801, 0.609801}

```

Questo risultato non ci sorprende, poichè è  $|f'(x)| \leq |\lambda|$  e stiamo considerando  $|\lambda| < 1$ . Inoltre, la composizione n-esima  $f(f(\dots f(x)))$  converge alla funzione costante  $f(x_*)$  dove  $x_* = 0.609801$ :

```

Clear[n]

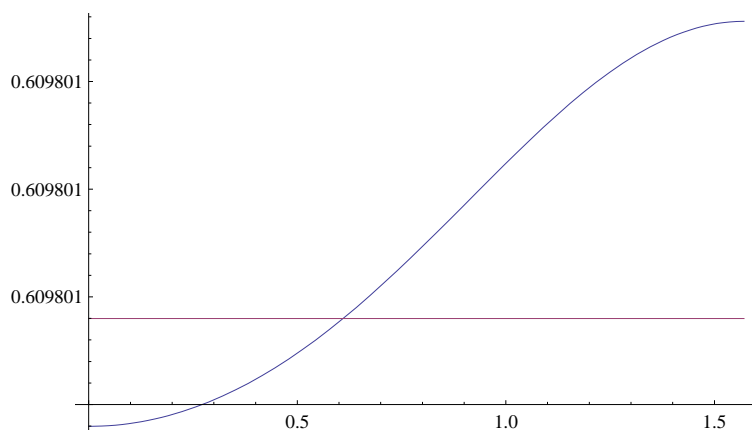
fn[x_, n_] := Nest[f, x, n]

Clear[plot]

plot[n_] := Plot[
  {
    fn[x, n],
    0.6098010708996272`
  },
  {x, 0,  $\pi/2$ }
]

plot[20]

```



Studiamo la ricorsione per  $|\lambda| > 1$ . Senza perdita di generalità consideriamo  $\lambda$  positivo. Quindi  $\lambda > 1$ . Iniziamo con  $\lambda = 2$  (per ora esaminiamo solo gli interi):

```

Clear[f]

f[x_] := ff[x, 2]

Clear[x0, iterazione]

```

```

x0[a_] := Random[
  (*tipo di numero (Real, Integer, Complex)*)
  Real,
  (*intervallo di appartenenza*)
  {-a, a}
]

iterazione[a_] := Table[
  Nest[
    f,
    x0[a],
    a],
  {20}
]

iterazione[10^2]

{0.93029, 0.93029, 0.93029, 0.93029, 0.93029, 0.93029, 0.93029, 0.93029, 0.93029, 0.93029,
  0.93029, 0.93029, 0.93029, 0.93029, 0.93029, 0.93029, 0.93029, 0.93029, 0.93029, 0.93029}

Plot[
  {
    f[x],
    x
  },
  {x, 0,  $\pi/2$ },
  PlotRange -> {0, 1}
]

FixedPoint[f, 1.5]

0.93029

Clear[n, fn, plot]

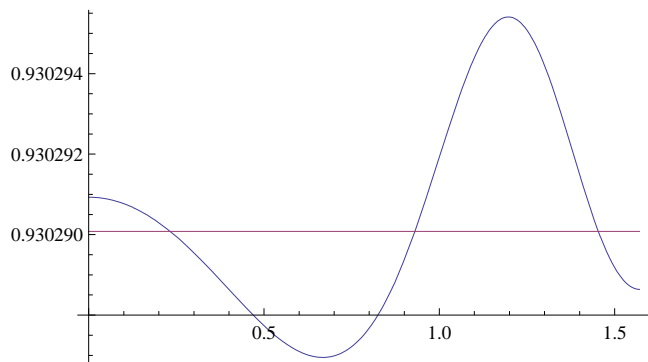
fn[x_, n_] := Nest[f, x, n]

plot[n_] := Plot[
  {
    fn[x, n],
    0.9302900815786262`
  },
  {x, 0,  $\pi/2$ }
]

```

Converge:

plot[20]



Passiamo a  $\lambda=3$

```
Clear[f, x0, plot, fn, iterazione]
```

```
f[x_] := ff[x, 3]
```

```
x0[a_] := Random[
  (*tipo di numero (Real, Integer, Complex)*)
  Real,
  (*intervallo di appartenenza*)
  {-a, a}
]
```

```
iterazione[a_] := Table[
  Nest[
    f,
    x0[a],
    a],
  {20}
]
```

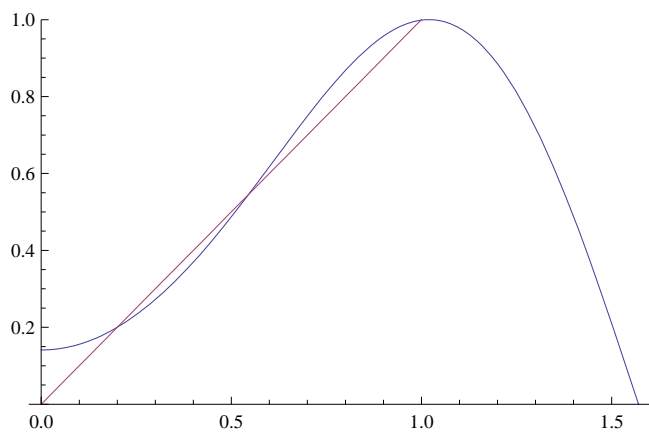
```
iterazione[10^2]
```

```
{0.998556, 0.200082, 0.200082, 0.998556, 0.998556, 0.998556,
 0.200082, 0.998556, 0.998556, 0.200082, 0.200082, 0.200082, 0.998556,
 0.200082, 0.200082, 0.998556, 0.200082, 0.200082, 0.998556, 0.200082}
```

```
FixedPoint[f, 1.4]
```

```
0.200082
```

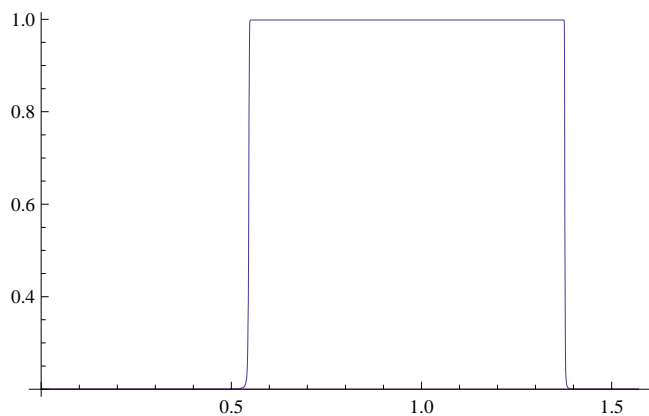
```
Plot[
{
  f[x],
  x
},
{x, 0,  $\pi/2$ },
PlotRange -> {0, 1}
]
```



```
fn[x_, n_] := Nest[f, x, n]
```

```
plot[n_] := Plot[
{
  fn[x, n],
  0.20008244817781284`
},
{x, 0,  $\pi/2$ }
]
```

```
plot[20]
```



```

In[46]:= ricorsione[f_, x0_, ini_, length_, xmin_: 0, xmax_:  $\pi/2$ ] := Block[
  {
    (*dichiaro le variabili locali*)
    start,
    orbit,
    plot,
    lines
  },
  (*definisco le variabili locali in funzione delle variabili di input*)
  start = Nest[
    f,
    N[x0],
    ini
  ];
  orbit = NestList[
    f,
    start,
    length
  ];
  plot = Plot[
    f[x],
    {x, xmin, xmax},
    DisplayFunction -> Identity
  ];
  lines = Line[Rest[Partition[Flatten[Transpose[{orbit, orbit}]], 2, 1]]];
  Show[plot, Graphics[
    {
      {Thickness[0.0001],
        PointSize[0.02],
        lines,
        Point[{start, f[start]}]
      },
      Line[{{xmin, xmin}, {xmax, xmax}}]}]
  ],
  Axes -> True,
  AxesLabel ->
  {
    "x", "y"
  },
  AxesOrigin ->
  {
    xmin,
    xmin
  },
  Ticks -> Automatic,
  TicksStyle -> Directive[
    Hue[5/6],
    11
  ],
  ImageSize ->
  {
    500, 500
  },
  DisplayFunction -> $DisplayFunction,
  PlotRange -> All
]
]

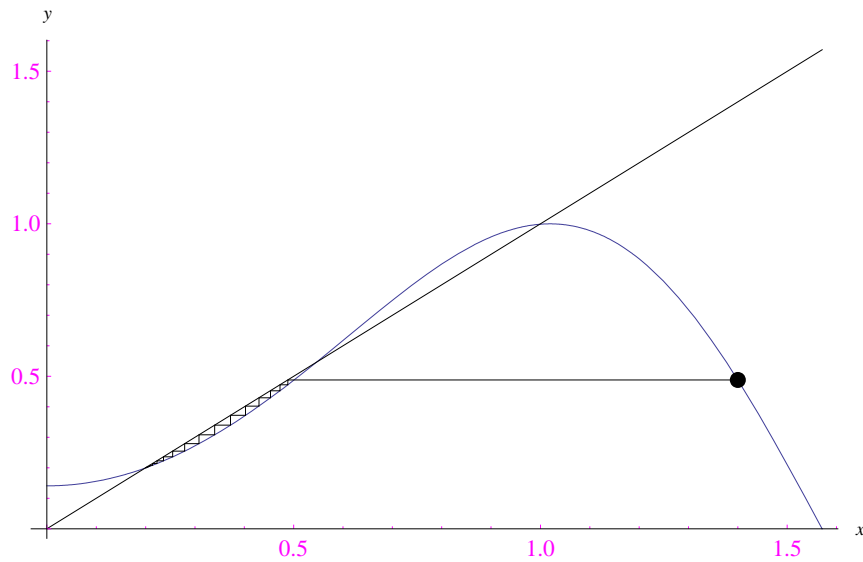
```



ri

```
In[22]:= SetAttributes[ricorsione, Listable]
```

```
ricorsione[f, 1.4, 0, 200, 0,  $\pi/2$ ]
```



Passiamo a  $\lambda=4$ :

```
Clear[f, x0, plot, fn, iterazione]
```

```
f[x_] := ff[x, 4]
```

```
x0[a_] := Random[
  (*tipo di numero (Real, Integer, Complex)*)
  Real,
  (*intervallo di appartenenza*)
  {-a, a}
]
```

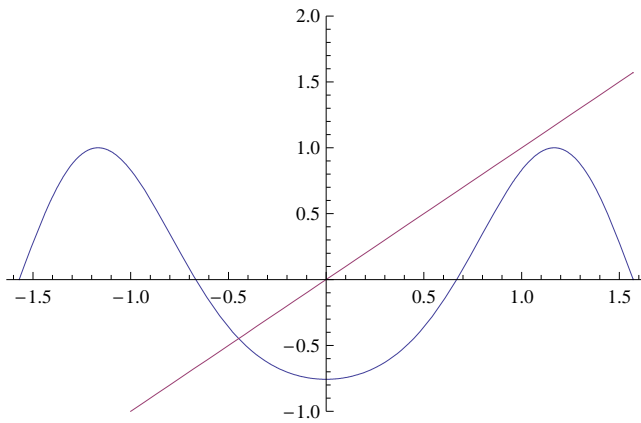
```
iterazione[a_] := Table[
  Nest[
    f,
    x0[a],
    a],
  {20}
]
```

```
iterazione[10^2]
```

```
{-0.752285, 0.0586571, 0.219282, -0.752288, 0.219287, -0.690817,
-0.752288, 0.219286, 0.0588073, 0.219286, 0.0586576, -0.690817, -0.691914,
0.219283, -0.690857, -0.693002, 0.219286, -0.752288, -0.690817, -0.75228}
```

```
FixedPoint[f, 1.4]
$Aborted

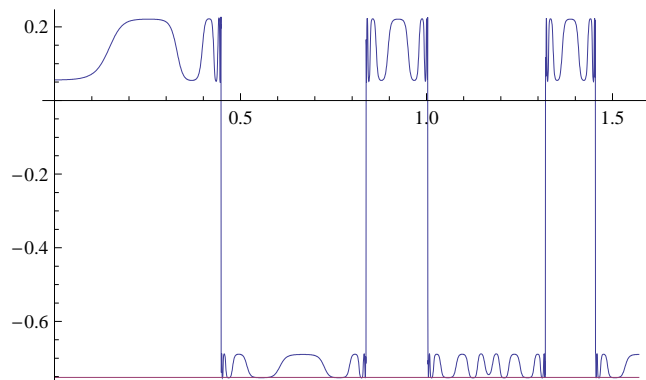
Plot[
{
  f[x],
  x
},
{x, - $\pi/2$ ,  $\pi/2$ },
PlotRange  $\rightarrow$  {-1, 2}
]
```



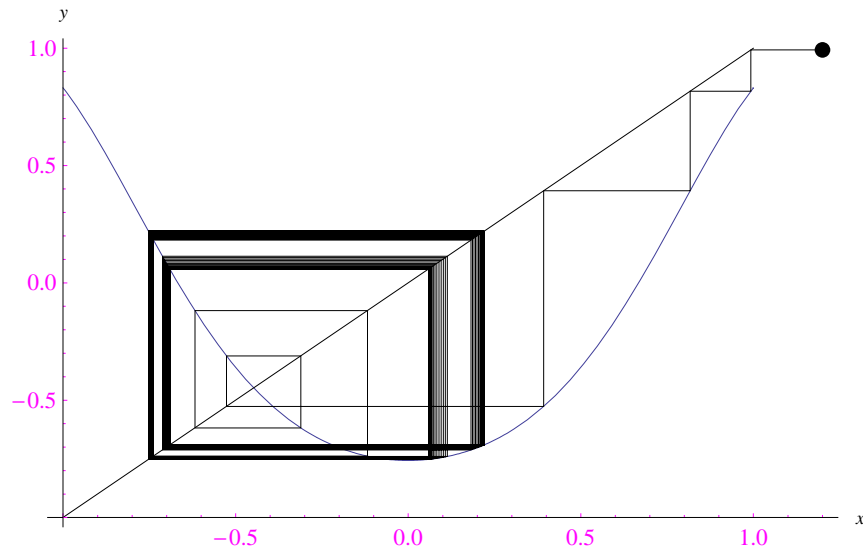
```
fn[x_, n_] := Nest[f, x, n]
```

```
plot[n_] := Plot[
{
  fn[x, n],
  -0.7522797548389334`
},
{x, 0,  $\pi/2$ }
]
```

```
plot[20]
```



```
ricorsione[f, 1.2, 0, 135, -1, 1]
```



Passiamo a  $\lambda=5$

```
Clear[f, x0, plot, fn, iterazione]
```

```
f[x_] := ff[x, 5]
```

```
x0[a_] := Random[
  (*tipo di numero (Real, Integer, Complex)*)
  Real,
  (*intervallo di appartenenza*)
  {-a, a}
]
```

```
iterazione[a_] := Table[
  Nest[
    f,
    x0[a],
    a],
  {20}
]
```

```
iterazione[10^2]
```

```
{-0.993787, -0.993787, 0.402276, -0.993788, 0.402276, 0.402275,
 0.402268, 0.40227, -0.993787, -0.993787, 0.402271, -0.993787, 0.402276,
-0.993788, 0.40227, 0.402269, -0.993788, -0.993787, -0.993787, -0.993789}
```

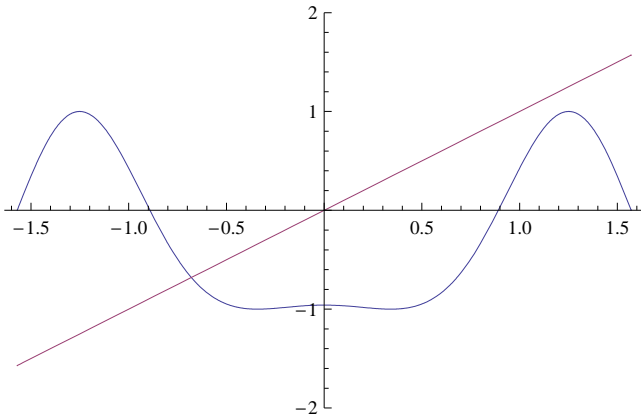
```
FixedPoint[f, 1.4]
```

```
$Aborted
```

```

Plot[
{
  f[x],
  x
},
{x, - $\pi/2$ ,  $\pi/2$ },
PlotRange -> {-2, 2}
]

```



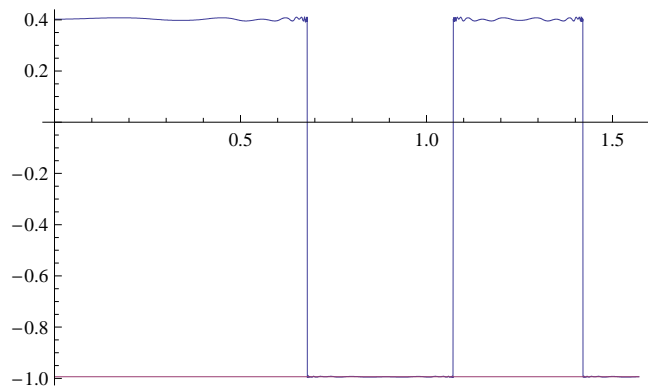
```
fn[x_, n_] := Nest[f, x, n]
```

```

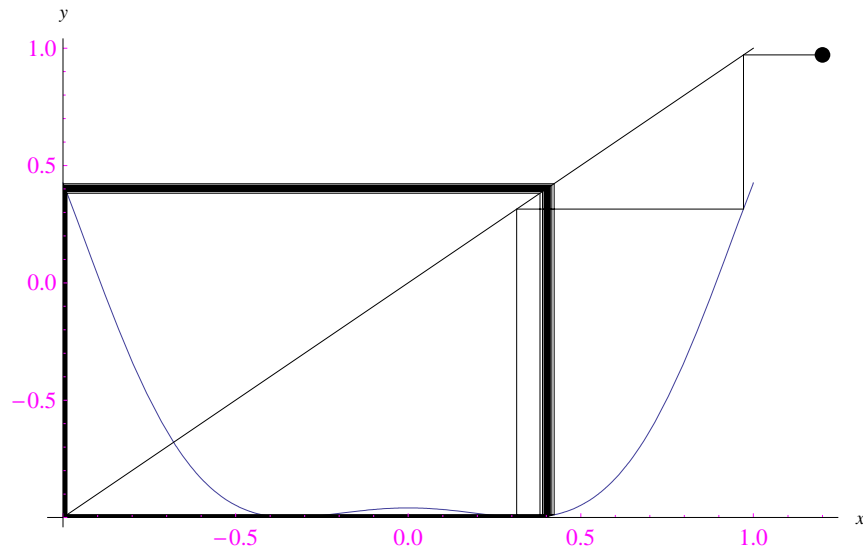
plot[n_] := Plot[
{
  fn[x, n],
  -0.9937890895868313`
},
{x, 0,  $\pi/2$ }
]

```

```
plot[20]
```



```
ricorsione[f, 1.2, 0, 135, -1, 1]
```



Passiamo a  $\lambda=6$

```
Clear[f, x0, plot, fn, iterazione]
```

```
f[x_] := ff[x, 6]
```

```
x0[a_] := Random[
  (*tipo di numero (Real, Integer, Complex)*)
  Real,
  (*intervallo di appartenenza*)
  {-a, a}
]
```

```
iterazione[a_] := Table[
  Nest[
    f,
    x0[a],
    a],
  {20}
]
```

```
iterazione[10^2]
```

```
{-0.567779, -0.993309, -0.99959, -0.533328, -0.994674, -0.995669,
-0.630209, -0.247144, -0.378778, -0.536019, -0.297798, -0.363875, -0.242201,
-0.107761, -0.54638, -0.983279, -0.247687, -0.775034, -0.84473, -0.52475}
```

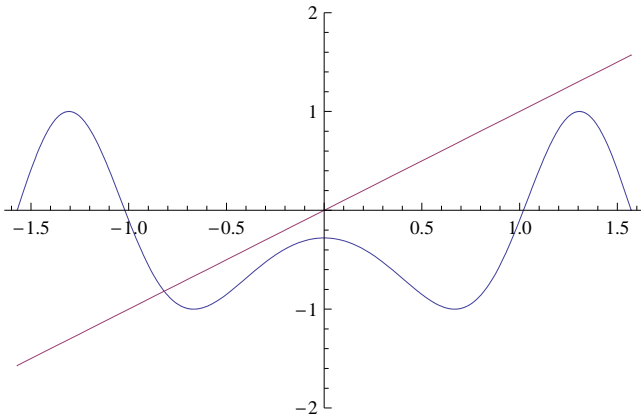
```
FixedPoint[f, 1.2]
```

```
$Aborted
```

```

Plot[
{
  ff[x, 6],
  x
},
{x, - $\pi/2$ ,  $\pi/2$ },
PlotRange -> {-2, 2}
]

```



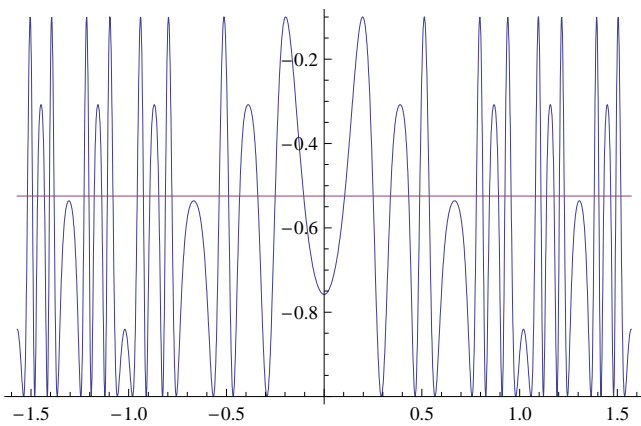
```
fn[x_, n_] := Nest[f, x, n]
```

```

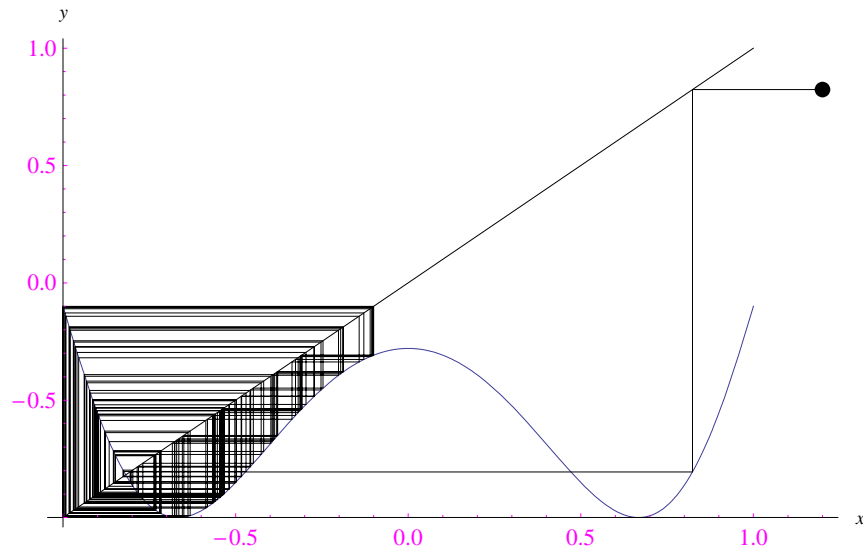
plot[n_] := Plot[
{
  fn[x, n],
  -0.5247496123248586`
},
{x, - $\pi/2$ ,  $\pi/2$ }
]

```

```
plot[4]
```



```
ricorsione[f, 1.2, 0, 135, -1, 1]
```



Passiamo a  $\lambda=7$

```
In[23]:= Clear[f, x0, plot, fn, iterazione]
```

```
In[24]:= f[x_] := ff[x, 7]
```

```
In[25]:= x0[a_] := Random[
  (*tipo di numero (Real, Integer, Complex)*)
  Real,
  (*intervallo di appartenenza*)
  {-a, a}
]
```

```
In[26]:= iterazione[a_] := Table[
  Nest[
    f,
    x0[a],
    a],
  {20}
]
```

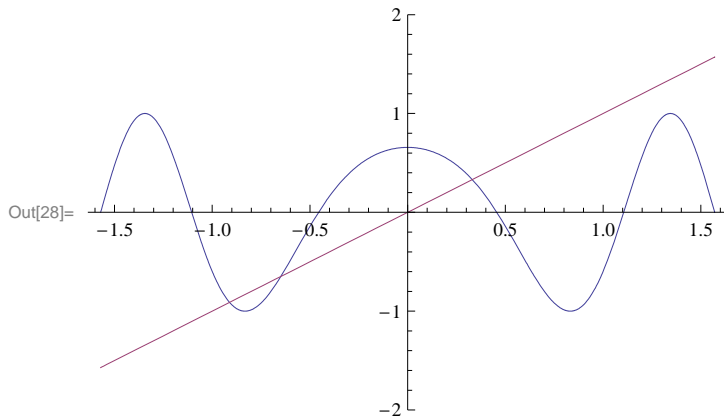
```
In[27]:= iterazione[10^2]
```

```
Out[27]:= {0.6091, -0.720033, -0.904035, 0.340021, -0.0265153, 0.627709,
  0.594451, -0.632695, 0.0790701, -0.574553, 0.194359, -0.598966, -0.233909,
  -0.674004, 0.554291, -0.807995, -0.676438, 0.599257, -0.694144, -0.671624}
```

```
FixedPoint[f, 1.2]
```

```
$Aborted
```

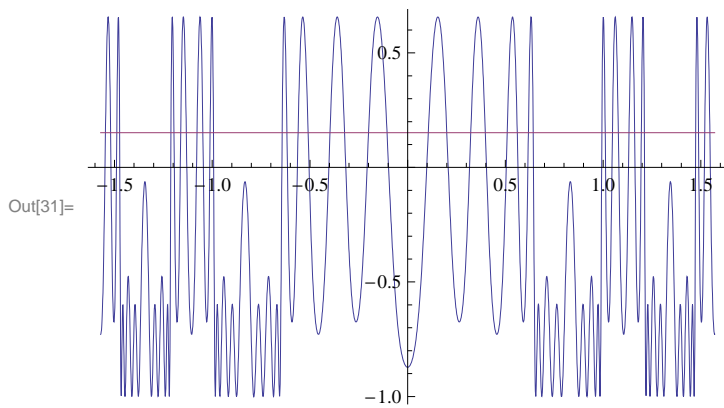
```
In[28]:= Plot[
  {
    ff[x, 7],
    x
  },
  {x, - $\pi/2$ ,  $\pi/2$ },
  PlotRange -> {-2, 2}
]
```



```
In[29]:= fn[x_, n_] := Nest[f, x, n]
```

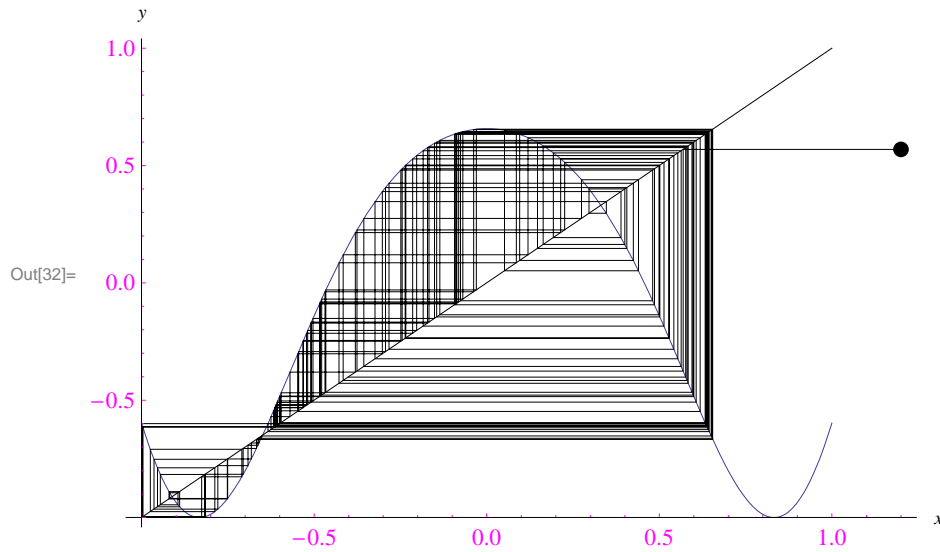
```
In[30]:= plot[n_] := Plot[
  {
    fn[x, n],
    0.151492364655506`
  },
  {x, - $\pi/2$ ,  $\pi/2$ }
]
```

```
In[31]:= plot[4]
```





```
In[32]:= ricorsione[f, 1.2, 0, 135, -1, 1]
```



```
In[51]:= ric[x0_] := ricorsione[f, x0, 0, 80, -1, 1]
```

```
In[53]:= movie = Table[
  ric[x0],
  {x0, 0.1, 0.8, 0.02}
];
```

```
In[54]:= SetDirectory["D:\\Siti\\extrabyte2\\Mathematica\\RICORSIONE"];
```

```
In[55]:= Export["xcvbb.gif", movie];
```

```
Clear[f, ff]
```

```
f[x_] := Sin[λ * Cos[x]]
```

```

OrbitDiagram[start_, end_, n_, init_, final_] := Show[
  Graphics[
    {PointSize[0.001],
      Table[
        Map[Point[{λ, #}] &,
          NestList[f, Nest[f, .5, init],
            final
          ]
        ],
        {λ, end, start, (start - end) / n}
      ]
    },
  PlotRange → Automatic,
  Axes → True,
  AxesLabel →
    {
      "λ"
    },
  AxesOrigin → {start, 0},
  ImageSize → {500, 500}
]

OrbitDiagram[1, 8, 100, 10, 320]

```

