

# Esercizio n. 126

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Calcolare il limite:

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} - \sin x - \cos x}{\ln(\sin 2x)} \quad (1)$$

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### Soluzione

Il limite si presenta nella forma indeterminata  $\frac{0}{0}$ :

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} - \sin x - \cos x}{\ln(\sin 2x)} = \frac{0}{0},$$

applicando la regola di De L'Hospital:

$$\begin{aligned} \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} - \sin x - \cos x}{\ln(\sin 2x)} &\stackrel{H}{=} \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{2 \frac{\cos 2x}{\sin 2x}} \\ &= \frac{1}{2} \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{\cot 2x} = \frac{0}{0} \stackrel{H}{=} \frac{1}{2} \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos x + \sin x}{2 \left(-\frac{1}{\sin^2 2x}\right)} \\ &= -\frac{1}{4} \lim_{x \rightarrow \frac{\pi}{4}} \sin^2 2x (\sin x + \cos x) = -\frac{1}{4} \left( \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) = -\frac{1}{2\sqrt{2}} \end{aligned}$$