Double Slit Experiment

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1 Introduction

The double slit experiment is probably one of the most controversial experiments in contemporary physics, as it seriously challenges "physical realism". What emerges is the dual particle and wave nature of matter. The corpuscular aspect takes shape in experiences in which a material particle (eg an electron) is observed individually. On the other hand, the wave behavior manifests itself in the typical interference and diffraction phenomena.

The wave-particle dualism reflects two opposing points of view inherent in the interpretation of physical phenomena. In fact, the latter are interpreted either as a manifestation of entities that can be schematised through a geometric point (Whitehead's simple location) or as a manifestation of a spatially extended field. From the point of view of the simple location, the phenomena are explained through the action of corpuscles or material particles, as for example in the case of gravitational attraction. In the "extended field" paradigm, on the other hand, physical processes can be interpreted through the action of a field in which energy propagates by waves (think, for example, of the electromagnetic field as the solution of Maxwell's equations).

In previous issues we have seen that material particles exhibit a dual corpuscular and wave nature. The corpuscular aspect takes shape in the experiences in which the particle is observed individually. On the other hand, the wave behavior manifests itself in the typical interference and diffraction phenomena.

Physical reality therefore exhibits a dualism that sees the paradigm of the simple location in contrast with that of the extended field. In the case under examination, the aforementioned dualism manifests itself in the wave-particle dichotomy, two aspects that coexist in a single entity that we call a "particle".

2 The experiment

To discuss the wave-particle duality we illustrate the experience of the [1] double slit. A source S emits an electron beam of energy E. The beam impinges on the plate S which has the slits numbered 1 and 2, as illustrated in fig. **??**. According to De Broglie hypothesis the single electron motion of the beam is equivalent to the propagation of a monochromatic plane wave:

$$\psi(\mathbf{x},t) = Ae^{\frac{i}{\hbar}(\mathbf{p}\cdot\mathbf{x}-Et)} \tag{1}$$

which affects the plate Σ_1, so that it is partly reflected and partly undergoes diffraction through the slits. If we denote the diffracted waves ψ_1 and ψ_2, the De Broglie wave in the region near the screen Σ_2 is:

$$\psi_{diff}(\mathbf{x},t) = \psi_1(\mathbf{x},t) + \psi_2(\mathbf{x},t) \tag{2}$$

For the statistical interpretation of Born, the probability density of finding the electron at a given point near the screen Σ_2 is

$$\rho_{diff}(\mathbf{x},t) = |\psi_1(\mathbf{x},t) + \psi_2(\mathbf{x},t)|^2, \tag{3}$$

while the probability current density is

$$\begin{aligned}\mathbf{j}_{diff}(\mathbf{x},t) &= \frac{\hbar}{2im}\left(\psi^*_{diff}\nabla\psi_{diff} - \psi_{diff}\nabla\psi^*_{diff}\right) \\ &= \frac{\hbar}{2im}\left[(\psi_1+\psi_2)^*\nabla(\psi_1+\psi_2) - (\psi_1+\psi_2)\nabla(\psi_1+\psi_2)^*\right]\end{aligned} \tag{4}$$

A detector R is placed on the screen Σ_2 and has a variable position identified by the abscissa x (cfr. fig. 1). If $\mathbf{n}$ is the unit vector of the external normal to Σ_2 and $d\sigma$ the surface element of R ovvero la sua sezione, or its section, the probability that an electron crosses $d\sigma$ in the unit time interval is

$$\Pi\left(x,t\right) = \mathbf{j}_{diff}\left(\mathbf{x},t\right) \cdot \mathbf{n} d\sigma, \tag{5}$$

where $\mathbf{j}_{diff}\left(\mathbf{x},t\right)$ is given by (4). Note that Π depends on the abscissa x and not on $\mathbf{x} = (x, y, z)$ as the current density must be calculated along S in which a system of abscissas has been fixed. From the experimental configuration it follows that the previous formula expresses the probability that R detects the passage of an electron in the unitary time interval.

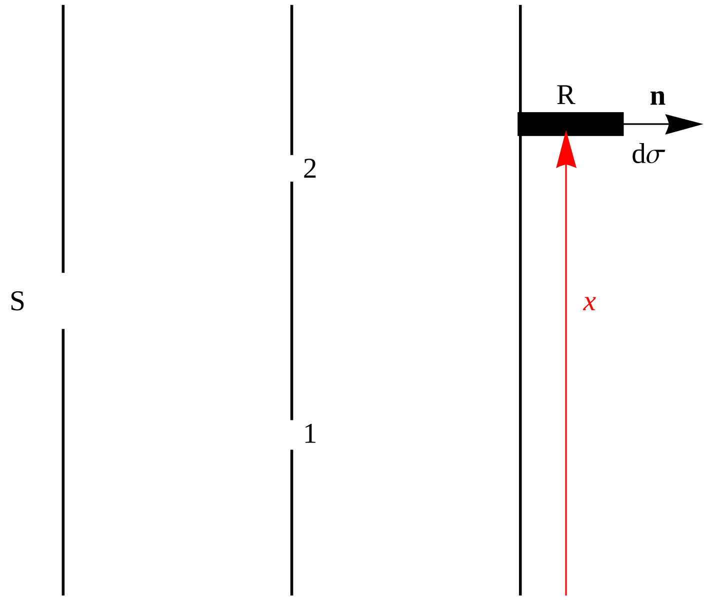

Figure 1: Illustration of the double slit experiment.

The time average of $\Pi\left(t\right)$ is

$$\bar{\Pi}\left(x\right) = \lim_{\tau \to +\infty} \frac{1}{\tau} \int_0^{\tau} \Pi\left(t\right) dt \tag{6}$$

If we limit ourselves $\tau \gg 1$, we have

$$\bar{\Pi}\left(x\right) \simeq \frac{1}{\tau} \int_0^{\tau} \Pi\left(t\right) dt, \tag{7}$$

which by the law of large numbers is

$$\bar{\Pi}\left(x\right) = \Phi\left(x\right),$$

where Φ is the flow of electrons in the unit of time through R, i.e. the number of electrons which in the unit of time cross the unitary section of R. As the position of R varies, we have Φ is a real function of the real variable x , i.e. $\Phi\left(x\right)$ whose graph is called an interference figure since it presents an alternation of maxima and minima typical of interference phenomena, as illustrated in fig. 2.

Let us now add two detectors R_1 and R_2 in correspondence with slits 1 and 2. In this way the number $N(t)$ of particles counted by R at time t is expressed as the sum of two contributions:

$$N\left(t\right) = N_1\left(t\right) + N_2\left(t\right), \tag{8}$$

where $N_k\left(t\right)$ is the number of particles counted by R_k at time t (for $k = 1, 2$). Considering how the detectors R_1 and R_2 are positioned, we have that $N_k\left(t\right)$ is the number of particles that have passed

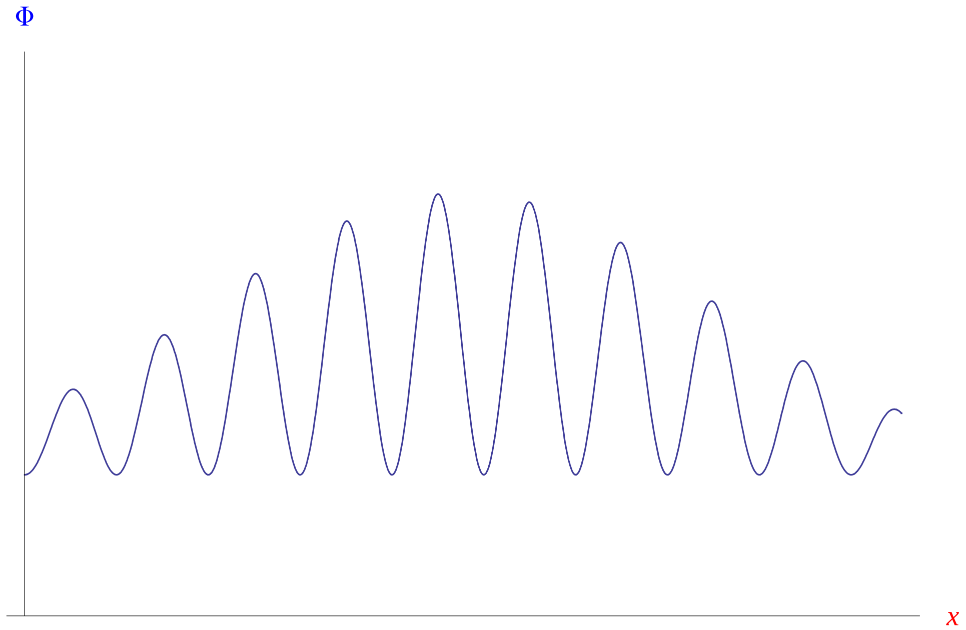

Figure 2: Interference pattern (graph of electron flow $\Phi(x)$).

through the k-th slit. Furthermore, it follows from (2) that the diffracted waves ψ_1 and ψ_2 give rise to probability currents.

$$\mathbf{j}_k(\mathbf{x},t) = \frac{\hbar}{2im}\left(\psi_k^* \nabla \psi_k - \psi_k \nabla \psi_k^*\right), \quad k = 1,2 \tag{9}$$

So the probability that R_k detects the passage of an electron in the unit time interval is:

$$\Pi_k(x_k,t) = \mathbf{j}_k(\mathbf{x},t) \cdot \mathbf{n} d\sigma_k, \tag{10}$$

where: x_k is the abscissa of R_k calculated with respect to an x axis arranged along Σ_1; $\mathbf{n}$ is the unit vector of the external normal tov Σ_1; $d\sigma_k$ is the section of R_k. Performing the time average we obtain the flow of electrons crossing R_k in the unit time:

$$\Phi_k = \lim_{\tau \to +\infty} \frac{1}{\tau} \int_0^\tau \Pi_k(x_k,t)\, dt \underset{\tau \gg 1}{\simeq} \frac{1}{\tau} \int_0^\tau \Pi_k(x_k,t)\, dt \tag{11}$$

From (8) we have that the total flux of the particles which cross the plate Σ_1 and which therefore reach the screen Σ_2 is:

$$\Phi_{tot}(x) = \Phi_1 + \Phi_2, \tag{12}$$

where now the abscissa x is measured along Σ_2. In the previous experimental setup (absence of R_k) we found:

$$\Phi(x) \underset{\tau \gg 1}{=} \frac{1}{\tau} \int_0^\tau \mathbf{j}_{diff}(\mathbf{x},t) \cdot \mathbf{n} d\sigma dt \tag{13}$$

Comparing with Φ_1 and Φ_2, given by

$$\Phi_1 \underset{\tau \gg 1}{=} \frac{1}{\tau} \int_0^\tau \mathbf{j}_1(\mathbf{x},t) \cdot \mathbf{n} d\sigma_1 dt, \qquad \Phi_2 \underset{\tau \gg 1}{=} \frac{1}{\tau} \int_0^\tau \mathbf{j}_2(\mathbf{x},t) \cdot \mathbf{n} d\sigma_2 dt,$$

we have

$$\Phi_{tot}(x) \neq \Phi(x),$$

since

$$\begin{aligned}\mathbf{j}_{diff}(\mathbf{x},t) &= \frac{\hbar}{2im}\left[(\psi_1+\psi_2)^*\nabla(\psi_1+\psi_2)-(\psi_1+\psi_2)\nabla(\psi_1+\psi_2)^*\right] \\ &= \frac{\hbar}{2im}[\psi_1^*\nabla(\psi_1+\psi_2)-\psi_1\nabla(\psi_1+\psi_2)^*+ \\ &+\psi_2^*\nabla(\psi_1+\psi_2)-\psi_2\nabla(\psi_1+\psi_2)^*] \\ &\neq \mathbf{j}_1(\mathbf{x},t)+\mathbf{j}_2(\mathbf{x},t)\end{aligned} \tag{14}$$

Plotting Φ_1 and Φ_2 as a function of x, we find a trend without oscillations. Precisely, the function $\Phi_1(x)$ will present a relative maximum (which is also the absolute maximum) in correspondence with the abscissa x_1 of the detector R_1, with a decreasing trend as we move away from the detector (along the plate Σ_1). An analogous behavior for the flow $\Phi_2(x)$ which will present a maximum in correspondence with R_2. It follows that also the sum $\Phi_1+\Phi_2=\Phi_{tot}$ will exhibit a trend without oscillations. Physically this is interpreted by asserting that the presence of the detectors has modified the phenomenon by destroying the interference pattern [1].

If we schematize the electron as a particle, we must necessarily assert that to reach R, the electron must necessarily pass through one of the two slits. And that's exactly what happens when we place the two detectors R_1 and R_2. However, in the absence of the latter, an interference pattern typical of wave phenomena is generated, and this destroys the particle character of the electron. But even the wave character cannot be applied without restrictions; in fact, it is sufficient to reposition the aforesaid detectors to cancel the interference pattern.

We conclude by observing that the wave-particle dualism reflects two opposing points of view inherent in the interpretation of [2] . physical phenomena. In fact, the latter are interpreted either as a manifestation of entities that can be schematised through a geometric point (Whitehead's *simple location)* or as a manifestation of a spatially extended field. From the point of view of the simple location, the phenomena are explained through the action of corpuscles or material particles, as for example in the case of gravitational attraction. In the "extended field" paradigm, on the other hand, physical processes can be interpreted through the action of a field in which energy propagates by waves (think, for example, of the electromagnetic field as the solution of Maxwell's equations).

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References

[1] Caldirola P. Cirelli R., Prosperi G.M. *Introduzione alla Fisica Teorica* Utet, 1987.

[2] Caldirola P. *Dalla microfisica alla macrofisica,* Mondadori 1978