
Calcolo di determinanti del second'ordine

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Esercizio 1 *Calcolare*

$$D = \begin{vmatrix} \psi & 1 \\ -1 & \psi \end{vmatrix}, \quad \psi = e^{i\varphi}, \quad \forall \varphi \in \mathbb{R} \quad (1)$$

Considerare il caso particolare $\varphi_0 = \frac{\pi}{3}$.

Soluzione

$$\begin{aligned} D &= \psi^2 + 1 = e^{2i\varphi} + 1 = \cos 2\varphi + i \sin 2\varphi + 1 \\ &= 2 \cos^2 \varphi - 1 + 2i \sin \varphi \cos \varphi + 1 \\ &= 2 \cos^2 \varphi + 2i \sin \varphi \cos \varphi \end{aligned} \quad (2)$$

Per $\varphi_0 = \frac{\pi}{3}$:

$$\cos \varphi_0 = \frac{1}{2}, \quad \sin \varphi_0 = \frac{\sqrt{3}}{2}, \quad (3)$$

onde

$$D = \frac{1}{2} + 2i \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{1}{2} + i \frac{\sqrt{3}}{2} \quad (4)$$