Calcolo di determinanti del second'ordine

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Esercizio 1 *Calcolare i seguenti determinanti:*

1. $D = \begin{vmatrix} a & z \\ z^* & b \end{vmatrix}$, *dove* $a, b \in \mathbb{R}$ *e* $z = c + id$, *mentre* z^* *è il complesso coniugato di* z.

2. $D(\alpha) = \begin{vmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{vmatrix}$

3. $D = \begin{vmatrix} z_1 & z_2 \\ z_2^* & z_1^* \end{vmatrix}$, *dove* $z_1 = \alpha + i\beta$, $z_2 = \gamma + i\delta$

4. $D(\alpha, \beta) = \begin{vmatrix} \sin\alpha & \cos\alpha \\ \sin\beta & \cos\beta \end{vmatrix}$

5. $D(\alpha, \beta) = \begin{vmatrix} \sin\alpha & \cos\alpha \\ \sin\beta & \cos\beta \end{vmatrix}$

6. $D(\alpha) = \begin{vmatrix} \tan\alpha & -1 \\ 1 & \tan\alpha \end{vmatrix}$

7. $D = \begin{vmatrix} 1+\sqrt{2} & 2-\sqrt{3} \\ 2+\sqrt{3} & 1-\sqrt{2} \end{vmatrix}$

8. $D(a, b) = \begin{vmatrix} 1 & \log_b a \\ \log_a b & 1 \end{vmatrix}$, $a, b > 0$, $a, b \neq 1$

9. $D = \begin{vmatrix} a+b & b+d \\ a+c & c+d \end{vmatrix}$, $a, b, c, d \in \mathbb{R}$

Soluzione

1.

$$\begin{aligned} D &= ab - (c+id)(c-id) \\ &= ab - \left(c^2 + d^2\right) \\ &\Longrightarrow D = ab - c^2 - d^2 \end{aligned} \tag{1}$$

2.

$$D(\alpha) = \sin^2\alpha + \cos^2\alpha = 1, \quad \forall \alpha \in \mathbb{R} \tag{2}$$

3.

$$D = z_1 z_1^* - z_2 z_2^* = |z_1|^2 - |z_2|^2 \tag{3}$$

Cioè

$$D = \alpha^2 - \beta^2 - \gamma^2 - \delta^2 \tag{4}$$

4.
$$D(\alpha,\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta \tag{5}$$

Dalle formule di addizione degli archi:

$$D(\alpha,\beta) = \sin(\alpha-\beta) \tag{6}$$

5.
$$D(\alpha,\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \tag{7}$$

Dalle formule di addizione degli archi:

$$D(\alpha,\beta) = \cos(\alpha+\beta) \tag{8}$$

6.
$$D(\alpha) = \tan^2\alpha + 1 = \frac{\sin^2\alpha}{\cos^2\alpha} + 1 \tag{9}$$

Cioè

$$D(\alpha) = \frac{1}{\cos^2\alpha} \tag{10}$$

7.
$$\begin{aligned} D &= \left(1+\sqrt{2}\right)\left(1-\sqrt{2}\right) + \left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right) \qquad (11) \\ &= 1-2-(4-3) = -1-1, \end{aligned}$$

onde

$$D = -2 \tag{12}$$

8.
$$D(a,b) = 1 - \log_b a \log_a b$$

Eseguendo il cambiamento di base $a \to b$:

$$\log_a b = \frac{\log_b b}{\log_b a} = \frac{1}{\log_b a},$$

per cui

$$D(a,b) = 0 \tag{13}$$

9.

$$\begin{aligned} D &= (a+b)(c+d) - (a+c)(b+d) \qquad (14) \\ &= ac + bc + ad + bd - ab - bc - ad - cd \\ &= a(c-b) - d(c-b) \end{aligned}$$

Quindi

$$D = (a-d)(c-b) \tag{15}$$