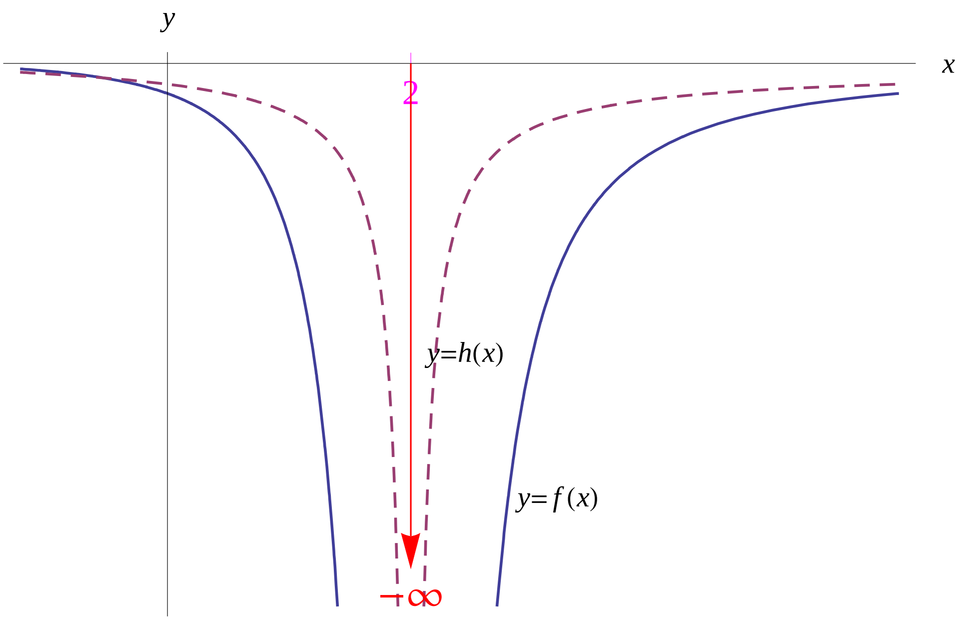
Matematica Open Source

$$\frac{d}{dx}f(x) \quad \sum_{k=0}^{+\infty} a_k \quad \int f(x)\,dx \quad \oint_{\Gamma} (X\,dx + Y\,dy + Z\,dz)$$

Generalizzazione del teorema dei carabinieri

Marcello Colozzo



Criterio 1 *Siano $f(x)$, $g(x)$ definite in $X \subseteq \mathbb{R}$ e regolari in $x_0 \in \mathcal{D}(X)$.*

- ***Ipotesi***

$$\lim_{x \to x_0} g(x) = +\infty$$
$$\exists I(x_0) \mid x \in X \cap I(x_0) - \{x_0\} \Longrightarrow g(x) \le f(x)$$

- ***Tesi***

$$\lim_{x \to x_0} f(x) = +\infty$$

Dimostrazione.

$$\lim_{x \to x_0} g(x) = +\infty \Longleftrightarrow (\forall \varepsilon > 0, \exists I_{\delta_\varepsilon}(x_0) \mid x \in X \cap I_{\delta_\varepsilon}(x_0) - \{x_0\} \Longrightarrow g(x) > \varepsilon$$

Per ipotesi:

$$\exists I(x_0) \mid x \in X \cap I(x_0) - \{x_0\} \Longrightarrow g(x) \le f(x)$$

Consideriamo il seguente intorno di x_0:

$$I_{\Delta_\varepsilon}(x_0) = I(x_0) \cap I_{\delta_\varepsilon}(x_0) = (x_0 - \Delta_\varepsilon, x_0 + \Delta_\varepsilon),$$

onde:

$$x \in X \cap I_{\Delta_\varepsilon}(x_0) - \{x_0\} \Longrightarrow f(x) \ge g(x) > \varepsilon$$

da cui:

$$\lim_{x \to x_0} f(x) = +\infty$$

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Criterio 2 *Siano $f(x)$, $h(x)$ definite in $X \subseteq \mathbb{R}$ e regolari in $x_0 \in \mathcal{D}(X)$.*

- ***Ipotesi***

$$\lim_{x \to x_0} h(x) = -\infty$$
$$\exists I(x_0) \mid x \in X \cap I(x_0) - \{x_0\} \Longrightarrow f(x) \le h(x)$$

Tesi

$$\lim_{x \to x_0} f(x) = -\infty$$

Dimostrazione. %

$$\lim_{x \to x_0} h(x) = -\infty \Longleftrightarrow (\forall \varepsilon > 0, \exists I_{\delta_\varepsilon}(x_0) \mid x \in X \cap I_{\delta_\varepsilon}(x_0) - \{x_0\} \Longrightarrow h(x) < -\varepsilon$$

Per ipotesi:

$$\exists I(x_0) \mid x \in X \cap I(x_0) - \{x_0\} \Longrightarrow f(x) \le h(x)$$

Consideriamo il seguente intorno di x_0:

$$I_{\Delta_\varepsilon}(x_0) = I(x_0) \cap I_{\delta_\varepsilon}(x_0) = (x_0 - \Delta_\varepsilon, x_0 + \Delta_\varepsilon),$$

onde:

$$x \in X \cap I_{\Delta_\varepsilon}(x_0) - \{x_0\} \Longrightarrow f(x) \le h(x) < -\varepsilon$$

da cui:

$$\lim_{x \to x_0} f(x) = -\infty$$

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