Calcolare i seguenti integrali indefiniti

1) $\int \frac{1}{2x}\,dx = \frac{1}{2}\int \frac{1}{x}\,dx = \frac{1}{2}\log|x| + c$

2) $\int 2x\,e^{x^2+2}\,dx = \int e^t\,dt = e^t + c = e^{x^2+2} + c$

pongo $x^2+2 = t$, differenziando ambo i membri: $2x\,dx = dt$

3) $\int (x-1)\sqrt{x}\,dx = \int \left(x^{3/2} - x^{1/2}\right)dx = \int x^{3/2}\,dx - \int x^{1/2}\,dx =$

$= \frac{x^{5/2}}{\frac{5}{2}} - \frac{x^{3/2}}{\frac{3}{2}} + c = \frac{2x^2\sqrt{x}}{5} - \frac{2x\sqrt{x}}{3} + c = 2x\sqrt{x}\left(\frac{3x-5}{15}\right) + c$

4) $\int (3x^2 - 6x + 3)\,dx = 3\int x^2\,dx - 6\int x\,dx + 3\int dx =$

$= 3\frac{x^3}{3} - 6\frac{x^2}{2} + 3x + c = x^3 - 3x^2 + 3x + c$

5) $\int \frac{x^4 + 4x^3 + 2}{x^3}\,dx = \int x\,dx + 4\int dx + 2\int x^{-3}\,dx =$

$= \frac{x^2}{2} + 4x + 2\frac{x^{-2}}{(-2)} + c = \frac{x^2}{2} + 4 - \frac{1}{x^2} + c.$

6) $\int \frac{3}{x+4}\,dx = 3\int \frac{dx}{x+4} = 3\int \frac{dt}{t} = 3\log|t| + c = 3\log|x+4| + c$

pongo $x+4 = t$

$dx = dx$

7) $\int \sin x\, e^{\cos x}\,dx = -\int e^t\,dt = -e^t + c = -e^{\cos x} + c$

pongo $\cos x = t$

$-\sin x\,dx = dt$

$\sin x\,dx = -dt$

8) $$\int \frac{(e^x-2)e^x}{e^x+1}\,dx = \int \frac{t-2}{t+1}\,dt = \int \frac{t+1-1-2}{t+1}\,dt =$$

pongo $e^x = t$

$e^x dx = dt$

$$= \int dt - 3\int \frac{dt}{t+1} = t - 3\log|t+1| + c = e^x - 3\log(e^x+1) + c$$

9) $$\int x \sin x\,dx = -x\cos x + \int \cos x\,dx = -x\cos x + \sin x + c$$

integro per parti

$f = x \quad g' = \sin x$

$f' = 1 \quad g = -\cos x$

10) $$\int x^2 \log x\,dx = \frac{x^3}{3}\log x - \int \frac{1}{x}\frac{x^3}{3}\,dx = \frac{x^3}{3}\log x - \frac{x^3}{9} + c$$

integro per parti

$f = \log x \quad g' = x^2$

$f' = \frac{1}{x} \quad g = \frac{x^3}{3}$

11) $$\int x^3 e^{2x}\,dx = x^3\frac{e^{2x}}{2} - \frac{3}{2}\int x^2 e^{2x}\,dx = \frac{x^3e^{2x}}{2} - \frac{3}{2}\left[\frac{x^2e^{2x}}{2} - \int x e^{2x}\,dx\right] =$$

integro per parti

$f = x^3 \quad g' = e^{2x}$

$f' = 3x^2 \quad g = \frac{e^{2x}}{2}$

integro per parti

$f = x^2 \quad g' = e^{2x}$

$f' = 2x \quad g = \frac{e^{2x}}{2}$

$$= \frac{x^3e^{2x}}{2} - \frac{3x^2e^{2x}}{4} + \frac{3}{2}\left[\frac{xe^{2x}}{2} - \int \frac{e^{2x}}{2}\,dx\right] = \frac{x^3e^{2x}}{2} - \frac{3x^2e^{2x}}{4} + \frac{3}{4}xe^{2x} - \frac{3}{8}e^{2x} + c.$$

integro per parti

$f = x \quad g'(x) = e^{2x}$

$f' = 1 \quad g(x) = \frac{e^{2x}}{2}$