

Serie di Fourier

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```
In[1]:= SetOptions[
  {
    Plot,
    ParametricPlot,
    ParametricPlot3D
  },
  TicksStyle -> Directive[
    Hue[5 / 6],
    9
  ]
];
```

Il calcolo dei coefficienti di Fourier di un'assegnata funzione periodica, potrebbe avere un peso computazionale notevole. *Mathematica* gestisce le serie di Fourier con il package **FourierSeries**

```
In[2]:= << FourierSeries`
```

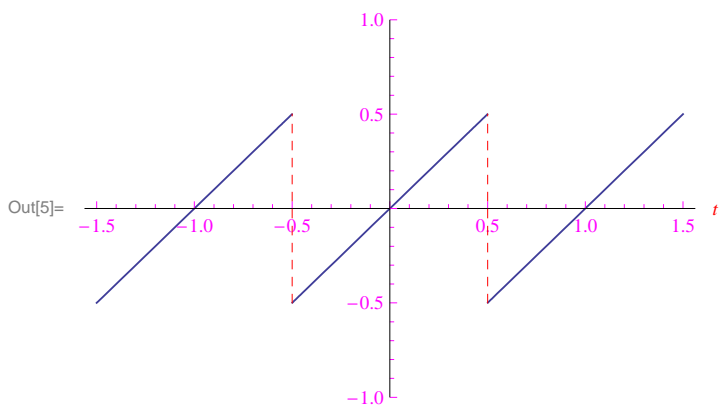
Supponiamo di avere la funzione periodica di periodo $T = 1$

```
In[3]:= f[t_] := Which[
  t >=  $\frac{-1}{2}$  && t <=  $\frac{1}{2}$ , t,
  t >=  $\frac{1}{2}$  && t <=  $\frac{3}{2}$ , t - 1,
  t >=  $\frac{-3}{2}$  && t <=  $\frac{-1}{2}$ , t + 1
];
```

```

In[5]:= plotf = Plot[
  f[t],
  {t, -3/2, 3/2},
  Exclusions -> {t == 1/2, t == -1/2},
  PlotRange -> {-1, 1},
  PlotStyle -> Thickness[0.003],
  AxesLabel ->
  {
    Style["t", Small, Red, Italic]
  },
  Epilog ->
  {
    Dashed,
    Red,
    Line[{{0.5, -0.5}, {0.5, 0.5}}],
    Line[{{-0.5, -0.5}, {-0.5, 0.5}}]
  }
]

```



```

In[11]:= ft[t_] = FourierTrigSeries[
  (*funzione*)
  f[t],
  (*variabile indipendente*)
  t,
  (*ordine della somma parziale*)
  6
]

```

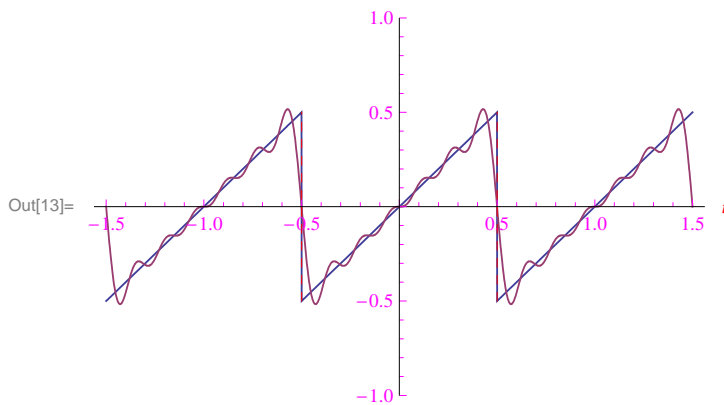
Out[11]=

$$\frac{\sin[2\pi t]}{\pi} - \frac{\sin[4\pi t]}{2\pi} + \frac{\sin[6\pi t]}{3\pi} - \frac{\sin[8\pi t]}{4\pi} + \frac{\sin[10\pi t]}{5\pi} - \frac{\sin[12\pi t]}{6\pi}$$

```

Plot[
{
  f[t], ft[t]
},
{t, -1.5, 1.5},
PlotRange -> {-1, 1},
PlotStyle -> Thickness[0.003],
AxesLabel ->
{
  Style["t", Small, Red, Italic]
},
Epilog ->
{
  Dashed,
  Red,
  Line[{{0.5, -0.5}, {0.5, 0.5}}],
  Line[{{-0.5, -0.5}, {-0.5, 0.5}}]
}
]

```



Se la funzione è troppo complicata, i coefficienti si calcolano numericamente:

```

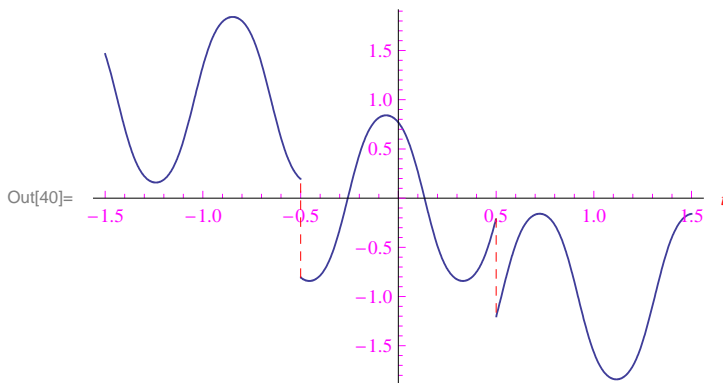
In[30]:= g[t_] := Which[
  t >= -1/2 && t <= 1/2, Sin[Cos[8 t + 1/2]],
  t >= 1/2 && t <= 3/2, Sin[Cos[8 t + 1/2]] - 1,
  t >= -3/2 && t <= -1/2, Sin[Cos[8 t + 1/2]] + 1
]

```

```

In[40]:= plotg = Plot[
  g[t],
  {t, -3/2, 3/2},
  Exclusions -> {t == 1/2, t == -1/2},
  PlotRange -> Automatic,
  PlotStyle -> Thickness[0.003],
  AxesLabel ->
  {
    Style["t", Small, Red, Italic]
  },
  Epilog ->
  {
    Dashed,
    Red,
    Line[{0.5, g[0.5]}, {0.5, Sin[Cos[8 * 0.5 + 1/2]] - 1}],
    Line[{-0.5, Sin[Cos[8 * (-0.5) + 1/2]]}, {-0.5, Sin[Cos[8 * (-0.5) + 1/2]] + 1}]
  }
]

```



```

In[49]:= gt[t_] = NFourierTrigSeries[
  (*funzione*)
  g[t],
  (*variabile indipendente*)
  t,
  (*ordine della somma parziale*)
  10
]

```

Out[49]=

$$\begin{aligned}
& -0.146026 + 0.762561 \cos[2\pi t] + 0.199471 \cos[4\pi t] - 0.0648093 \cos[6\pi t] + \\
& 0.0303291 \cos[8\pi t] - 0.0198239 \cos[10\pi t] + 0.0132861 \cos[12\pi t] - \\
& 0.0100702 \cos[14\pi t] + 0.00758577 \cos[16\pi t] - 0.00594826 \cos[18\pi t] + \\
& 0.00479846 \cos[20\pi t] - 0.326301 \sin[2\pi t] - 0.173441 \sin[4\pi t] + 0.0898221 \sin[6\pi t] - \\
& 0.0187531 \sin[8\pi t] + 0.0371297 \sin[10\pi t] - 0.0319734 \sin[12\pi t] + \\
& 0.0271795 \sin[14\pi t] - 0.023854 \sin[16\pi t] + 0.0212027 \sin[18\pi t] - 0.0190738 \sin[20\pi t]
\end{aligned}$$

Approssimazione valida solo nell'intervallo di periodicit  [-1/2, 1/2]

```
In[52]:= Plot[
  {
    g[t], gt[t]
  },
  {t, -3/2, 3/2},
  PlotRange -> Automatic,
  Exclusions -> {t == 1/2, t == -1/2},
  PlotStyle -> Thickness[0.003],
  AxesLabel ->
  {
    Style["t", Small, Red, Italic]
  }
]
```

