Serie di Fourier

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In[1]:=
```
SetOptions[
  {
   Plot
  },
  TicksStyle -> Directive[
    Hue[5 / 6],
    9
   ]
 ];
```

In[2]:=
```
Needs["FourierSeries`"]
```

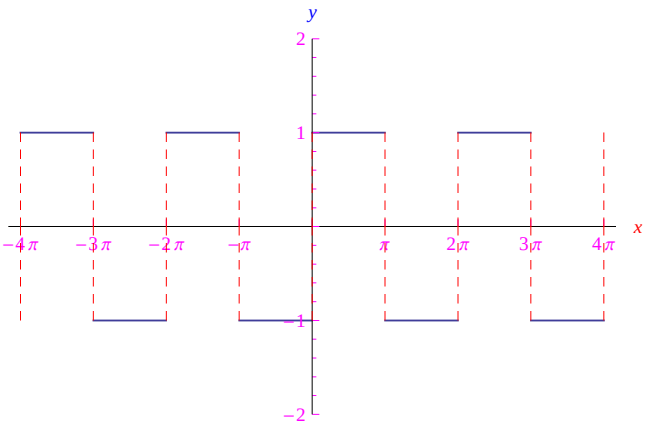
In[3]:=
$$f[x\_] := \frac{x - 2\pi\,\text{Round}\left[\frac{x}{2\pi}\right]}{\text{Abs}\left[x - 2\pi\,\text{Round}\left[\frac{x}{2\pi}\right]\right]}$$

In[4]:=
```
Table[x == k, {k, -4 π, 4 π, 2 π}]
```

Out[4]=
$$\{x == -4\pi,\ x == -2\pi,\ x == 0,\ x == 2\pi,\ x == 4\pi\}$$

In[5]:=
```
ondaquadra = Plot[
  f[x],
  {x, -4 π, 4 π},
  PlotRange → {-2.0, 2.0},
  Exclusions → Table[x == k, {k, -4 π, 4 π, π}],
  PlotStyle → Thickness[0.003],
  AxesLabel →
   {
    Style["x", Small, Red, Italic],
    Style["y", Small, Blue, Italic]
   },
  Ticks -> {
    Table[k, {k, -4 π, 4 π, π}]
   },
  Epilog → {
    Red,
    Dashed,
    Table[Line[{{k, 0}, {k, -1}}], {k, -4 π, 4 π, π}],
    Table[Line[{{k, 0}, {k, 1}}], {k, -4 π, 4 π, π}]
   }
 ]
```

Out[5]=

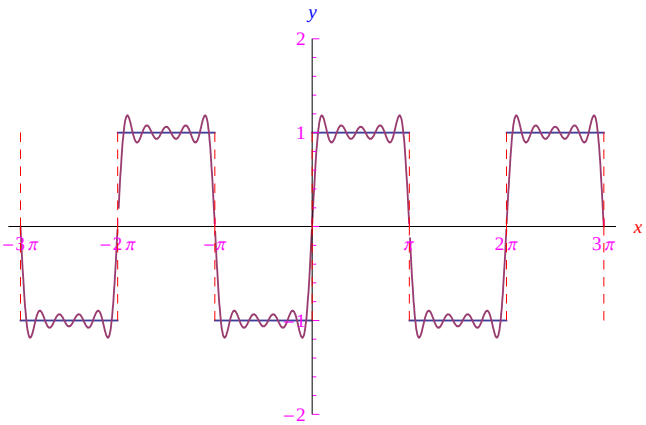


In[6]:=
```
S10[x_] = FourierTrigSeries[
  (*funzione*)
  f[x],
  (*variabile indipendente*)
  x,
  (*ordine*)
  10,
  (*periodo T=2π*)
  FourierParameters → {0, 1/(2 π)}
 ]
```

Out[6]=
$$\frac{4\sqrt{\frac{2}{\pi}}\,\text{Sin}[x] + \frac{4}{3}\sqrt{\frac{2}{\pi}}\,\text{Sin}[3x] + \frac{4}{5}\sqrt{\frac{2}{\pi}}\,\text{Sin}[5x] + \frac{4}{7}\sqrt{\frac{2}{\pi}}\,\text{Sin}[7x] + \frac{4}{9}\sqrt{\frac{2}{\pi}}\,\text{Sin}[9x]}{\sqrt{2\pi}}$$

In[7]:=
```
ondaquadra2 = Plot[
  {f[x], S10[x]},
  {x, -3 π, 3 π},
  PlotRange → {-2.0, 2.0},
  Exclusions → Table[x == k, {k, -4 π, 4 π, π}],
  PlotStyle → Thickness[0.003],
  AxesLabel →
   {
    Style["x", Small, Red, Italic],
    Style["y", Small, Blue, Italic]
   },
  Ticks -> {
    Table[k, {k, -4 π, 4 π, π}]
   },
  Epilog → {
    Red,
    Dashed,
    Table[Line[{{k, 0}, {k, -1}}], {k, -4 π, 4 π, π}],
    Table[Line[{{k, 0}, {k, 1}}], {k, -4 π, 4 π, π}]
   }
 ]
```

Out[7]=



In[8]:=
```
a[n_] = FourierCosCoefficient[f[x], x, n]
```

Out[8]= 0

In[9]:=
```
b[n_] = FourierSinCoefficient[f[x], x, n] // Simplify
```

Out[9]= $-\frac{2\left(-1+(-1)^{n}\right)}{n\,\pi}$

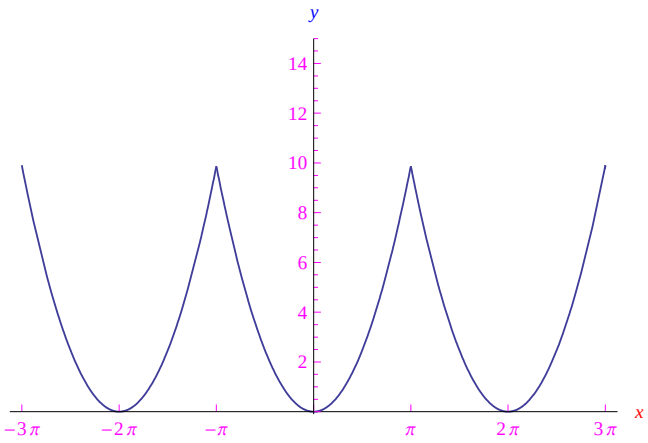
In[10]:=
```
Clear[f, a, b]
```

In[11]:=
```
f[x_] := (x - 2 π Round[x/(2 π)])^2
```

In[12]:=
```
plotf = Plot[
  f[x],
  {x, -3 π, 3 π},
  PlotRange → {-0.1, 15},
  PlotStyle → Thickness[0.003],
  Ticks →
   {
    Table[k, {k, -4 π, 4 π, π}]
   },

  AxesLabel →
   {
    Style["x", Small, Red, Italic],
    Style["y", Small, Blue, Italic]
   }
 ]
```

Out[12]=

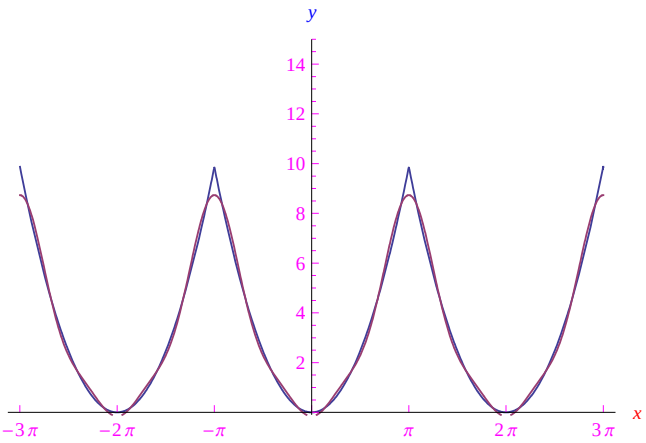


In[13]:=
```
S3[x_] = FourierTrigSeries[
   (*funzione*)
   f[x],
   (*variabile indipendente*)
   x,
   (*ordine*)
   3,
   (*periodo T=2π*)
   FourierParameters → {0, 1/(2 π)}
  ] // Simplify
```

Out[13]= $\frac{\pi^2}{3} - 4\,\text{Cos}[x] + \text{Cos}[2x] - \frac{4}{9}\,\text{Cos}[3x]$

In[14]:=
```
plotf = Plot[
  {
   f[x],
   S3[x]
  },
  {x, -3 π, 3 π},
  PlotRange → {-0.1, 15},
  PlotStyle → Thickness[0.003],
  Ticks →
   {
    Table[k, {k, -4 π, 4 π, π}]
   },
  AxesLabel →
   {
    Style["x", Small, Red, Italic],
    Style["y", Small, Blue, Italic]
   }
 ]
```

Out[14]=



In[15]:=
```
a[n_] = FourierCosCoefficient[f[x], x, n]
```

Out[15]=
$$\frac{(-1)^n}{n^2 \pi^2}$$

In[16]:=
```
b[n_] = FourierSinCoefficient[f[x], x, n]
```

Out[16]=
```
0
```

In[17]:=
```
a0 = FourierCosCoefficient[f[x], x, 0]
```

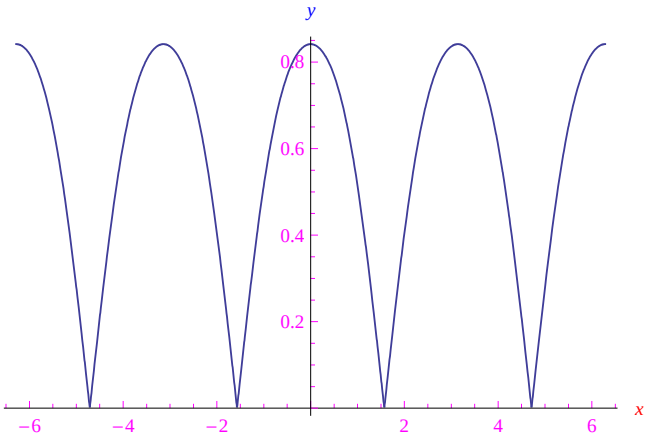
Out[17]=
$$\frac{1}{12}$$

In[18]:=
```
Clear[f]
```

In[19]:=
```
f[x_] := Sin[Cos[x - π Round[x / π]]]
```

In[20]:=
```
Plot[
 f[x],
 {x, -2 π, 2 π},
 PlotStyle → Thickness[0.003],
 AxesLabel →
  {
   Style["x", Small, Red, Italic],
   Style["y", Small, Blue, Italic]
  }
]
```

Out[20]=

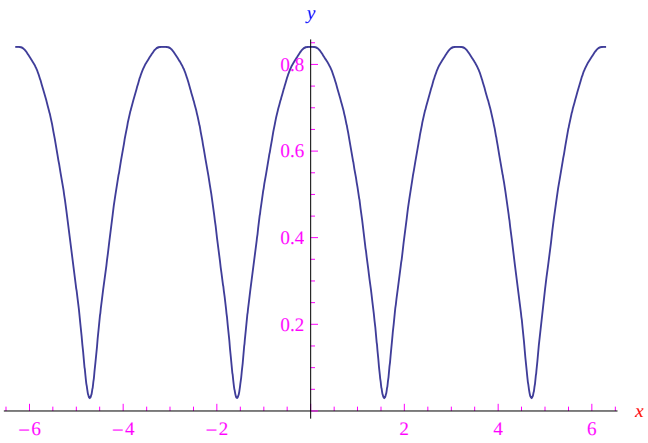


In[21]:=
```
S10[x_] = NFourierTrigSeries[
  f[x],
  x,
  10,
  FourierParameters → {0, 1 / π}
 ]
```

Out[21]=
$$\frac{1}{\sqrt{\pi}}\,(1.00792 + 0.608809\,\text{Cos}[2\,x] - 0.169627\,\text{Cos}[4\,x] + 0.0670677\,\text{Cos}[6\,x] - 0.0364906\,\text{Cos}[8\,x] + 0.0230494\,\text{Cos}[10\,x] - 0.0158994\,\text{Cos}[12\,x] + 0.0116354\,\text{Cos}[14\,x] - 0.00888602\,\text{Cos}[16\,x] + 0.00700912\,\text{Cos}[18\,x] - 0.00567054\,\text{Cos}[20\,x] + 0.\,\text{Sin}[2\,x] + 0.\,\text{Sin}[4\,x] + 0.\,\text{Sin}[6\,x] + 0.\,\text{Sin}[8\,x] + 0.\,\text{Sin}[10\,x] + 0.\,\text{Sin}[12\,x] + 0.\,\text{Sin}[14\,x] + 0.\,\text{Sin}[16\,x] + 0.\,\text{Sin}[18\,x] + 0.\,\text{Sin}[20\,x])$$

In[22]:=
```
Plot[
 S10[x],
 {x, -2 π, 2 π},
 PlotStyle → Thickness[0.003],
 AxesLabel →
  {
   Style["x", Small, Red, Italic],
   Style["y", Small, Blue, Italic]
  }
]
```

Out[22]=



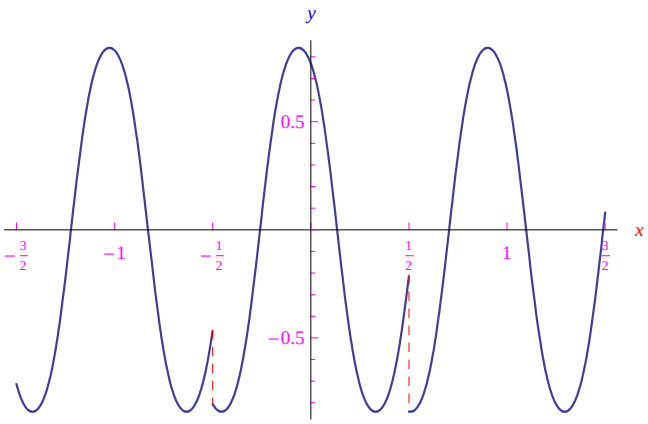
In[35]:=
```
Clear[g]
```

In[40]:=
```
g[t_] := Which[
  t ≥ -1/2 && t ≤ 1/2, Sin[Cos[8 * t + 1/2]],
  t ≥ -3/2 && t ≤ -1/2, -Sin[Cos[8 * (t - 1) + 1/2]],
  t ≥ 1/2 && t ≤ 3/2, -Sin[Cos[8 * (t + 1) + 1/2]]
 ]
```

In[59]:=
```
Plot[
 g[t],
 {t, -3 / 2, 3 / 2},
 Exclusions → {x = -1 / 2, x = 1 / 2},
 PlotStyle → Thickness[0.0034],
 AxesLabel →
  {
   Style["x", Small, Red, Italic],
   Style["y", Small, Blue, Italic]
  },
 Epilog → {
   Red,
   Dashed,
   Line[{{-1 / 2, -Sin[Cos[8 * (-1 / 2 - 1) + 1/2]]}, {-1 / 2, Sin[Cos[8 * (-1 / 2) + 1/2]]}}],
   Line[{{1 / 2, Sin[Cos[8 * (-1 / 2 + 1) + 1/2]]}, {1 / 2, Sin[Cos[8 * (1 / 2 - 1) + 1/2]]}}]
  },
 Ticks →
  {
   Table[k, {k, -3 / 2, 3 / 2, 1 / 2}]

  }
]
```

Out[59]=



In[61]:=
```
gt[t_] = NFourierTrigSeries[
  (*funzione*)
  g[t],
  t,
  10
 ]
```

Out[61]=
$$-0.146026 + 0.762561\cos[2\pi t] + 0.199471\cos[4\pi t] - 0.0648093\cos[6\pi t] + 0.0303291\cos[8\pi t] - 0.0198239\cos[10\pi t] + 0.0132861\cos[12\pi t] - 0.0100702\cos[14\pi t] + 0.00758577\cos[16\pi t] - 0.00594826\cos[18\pi t] + 0.00479846\cos[20\pi t] - 0.326301\sin[2\pi t] - 0.173441\sin[4\pi t] + 0.0898221\sin[6\pi t] - 0.0187531\sin[8\pi t] + 0.0371297\sin[10\pi t] - 0.0319734\sin[12\pi t] + 0.0271795\sin[14\pi t] - 0.023854\sin[16\pi t] + 0.0212027\sin[18\pi t] - 0.0190738\sin[20\pi t]$$

In[62]:=
```
Plot[
 {g[t], gt[t]},
 {t, -3 / 2, 3 / 2},
 Exclusions → {x = -1 / 2, x = 1 / 2},
 PlotStyle → Thickness[0.0034],
 AxesLabel →
  {
   Style["x", Small, Red, Italic],
   Style["y", Small, Blue, Italic]
  },
 Epilog → {
   Red,
   Dashed,
   Line[{{-1 / 2, -Sin[Cos[8 * (-1 / 2 - 1) + 1/2]]}, {-1 / 2, Sin[Cos[8 * (-1 / 2) + 1/2]]}}],
   Line[{{1 / 2, Sin[Cos[8 * (-1 / 2 + 1) + 1/2]]}, {1 / 2, Sin[Cos[8 * (1 / 2 - 1) + 1/2]]}}]
  },
 Ticks →
  {
   Table[k, {k, -3 / 2, 3 / 2, 1 / 2}]

  }
]
```

Out[62]=