ESERCIZI DI ANALISI I

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Determinare l'insieme in cui sono definite le seguenti funzioni:

(1) $\sqrt{\log_{1/2}\left(\operatorname{arctg}\dfrac{x-\pi}{x-4}\right)}$

(2) $\sqrt{\log_{\frac{\pi}{6}}\left|\operatorname{arcsen}\left(\dfrac{2x^3-x}{2}\right)\right|-1}$

(3) $\operatorname{tg}\sqrt{1-\log^2_{\frac{\pi}{4}}(x+1)}$

(4) $\sqrt{\dfrac{\sqrt{\operatorname{sen}^2 x-\operatorname{sen} x}-\operatorname{sen} x}{\operatorname{tg}^2 x-3}}$

(5) $\sqrt{100^{\operatorname{sen}^2 x-\cos^2 x}-10^{\operatorname{tg}^2 x-2}}$

(6) $\left[\dfrac{\sqrt{\sqrt{2x^2-16x+28}-x}}{\pi-2\operatorname{arctg}\left(\sqrt{3^{\frac{1}{x}}-2^{\frac{1}{x}}}\right)}\right]^{\frac{x}{1-\cos x}}$

(7) $\operatorname{arctg}\sqrt{\dfrac{|x^2-2|+1}{x^2-1}}$

(8) $\log\left(\dfrac{|\cos x|-\operatorname{sen} 2x}{\sqrt{\pi^2-9\arctan^2\left|\frac{x\sqrt{3}}{x+1}\right|}}\right)$

(9) $\sqrt{\dfrac{|\operatorname{tg} x|-|\operatorname{sen} x|}{x-\mathrm{e}\log x}}$

(10) $\dfrac{\sqrt{\frac{1}{2}-\log_3\left(\frac{1}{2}\operatorname{tg} x+\operatorname{sen} x\right)}+\sqrt{\pi^2-4x^2}}{\operatorname{arcsen}(\sqrt{x^2-x}-|x|)}$

(11) $\arccos\Big(\log_2(-\cos x)-\log_4\Big(\operatorname{sen} x+\dfrac{1}{2}\Big)\Big)$

(12) $\sqrt[2k]{\log_{\operatorname{sen}^{k+1} x}(\log(x-3))}$

(13) $\sqrt{1-\log_3\left(\dfrac{3-|x|}{\sqrt{x+2}}-\sqrt{x}\right)}$

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(14) $\operatorname{arcsen}\sqrt{1-\log^2_{\frac12}(\operatorname{sen} x)}$

(15) $\sqrt{\arccos\log_2(\operatorname{sen}\mathrm{e}^{\,x})-\dfrac{2\pi}{3}}$

(16) $\sqrt{1+\log_{2/\pi}\left(\arccos\dfrac{x}{x-1}\right)}$

(17) $\sqrt{\log_{\frac{\pi}{2}}(2\operatorname{arcsen} x)-1}$

(18) $\log_2\operatorname{arcsen}\dfrac{3x^2-x^3}{2}+\sqrt{\dfrac{3\operatorname{tg}^2(\frac{x}{2})-1}{\cos x-\operatorname{sen} x+1}}$

(19) $\sqrt{\log_3\big(\sqrt{x^2-1}-3x+4\big)+\log_{\frac19}(x^2+2x+1)}$

(20) $\dfrac{\big[(x-9)(\operatorname{sen} x+\cos x-1)\big]^{7/2}}{\sqrt{(\sqrt3+2)\cos x-\operatorname{sen} x-1}}$

(21) $\log\Big(\dfrac{|\operatorname{tg} x|-|\operatorname{sen} x|}{\sqrt{1+x^2}-x}\Big)$

(22) $\sqrt{\log_{1/2}\left(\dfrac{2+\cos x}{4-2\operatorname{sen} x-\cos x}\right)}$

Calcolare i seguenti limiti:

(23) $\displaystyle\lim_{x\to0}\frac1x\left[\sqrt[3]{\frac{1-\sqrt{1-x}}{\sqrt{1+x}-1}}-1\right]$

(24) $\displaystyle\lim_{x\to0}\frac{\log(1+x)-\log(1-\operatorname{sen} x)}{x+\operatorname{sen} x}$

(25) $\displaystyle\lim_{x\to0}\frac{\operatorname{arctg}^2\big[2\operatorname{sen} x-\log(1+x)\big]}{(2^x-1)\big(\sqrt{1+\operatorname{tg} x}-1\big)}$

(26) $\displaystyle\lim_{x\to0}\left(\frac{1+\operatorname{sen}^2x}{1-x}\right)^{\frac{1}{\operatorname{tg} x}}$

(27) $\displaystyle\lim_{x\to0}\frac{[\log(1+x\operatorname{sen} x)]\cdot\operatorname{arctg}^2(\mathrm{e}^{\,x}-1)}{\sqrt[4]{1+3x^4}-1}$

(28) $\displaystyle\lim_{x\to0^+}\frac{1-\cos x-\operatorname{sen} 2x}{\pi^2-9\operatorname{arctg}^2\left(\frac{\sqrt3}{1+x}\right)}$

(29) $\displaystyle\lim_{x\to0}\frac{\big(1+\operatorname{sen} x+\operatorname{sen}^2x\big)^{\frac1x}-\big(1+\operatorname{sen} x\big)^{\frac1x}}{x}$

(30) $\displaystyle\lim_{x\to+\infty}\frac{x^4\operatorname{sen}^2(\pi-2\operatorname{arctg} x)}{3+x^2}$

(31) $\displaystyle\lim_{x\to0}\frac{4^{\arccos^2\frac{1}{1+x^2}}-1}{\log_4(1+x)}$

(32) $\displaystyle\lim_{x\to 0}\frac{4\operatorname{sen} x-2\operatorname{tg}^3\sqrt[3]{3x}}{\cos x+\operatorname{sen} x-1}$

(33) $\displaystyle\lim_{x\to 0^+}\left[\frac{\pi}{2}+\operatorname{tg} x-\operatorname{arctg}\frac{1}{x}\right]^{\frac{1}{\log x}}$

(34) $\displaystyle\lim_{x\to 0^+}\frac{1}{x}\left[\left(-\frac{\log(1-x)}{\log(1+x)}\right)^{\alpha}-1\right],\ \alpha\in\mathbb{R}$

Derivare le seguenti funzioni:

(35) $\dfrac{1}{x}$ **(36)** $\dfrac{1}{x^2}$ **(37)** $\dfrac{1}{x^n}$

(38) $x^3\sqrt[3]{x^2}$ **(39)** $\dfrac{1}{\sqrt[4]{x^3}}$ **(40)** $\dfrac{1}{x^2\sqrt[3]{x^2}}$

(41) $5\operatorname{sen} x+4\operatorname{tg} x$ **(42)** $\sqrt{x}+2\cos x$ **(43)** $5^x+\operatorname{arcsen} x+\log_3 x$

(44) $\dfrac{1}{x^2}+\operatorname{arctg} x-\sqrt[5]{x}$ **(45)** $\operatorname{tg} x-2\operatorname{arcsen} x+\dfrac{1}{\sqrt{x}}$

(46) $7\arccos x-10^x+\operatorname{cotg} x+x\sqrt[3]{x}$ **(47)** $\operatorname{sen} x\operatorname{arcsen} x$

(48) $x\log_3 x$ **(49)** $(1+x^2)\operatorname{arctg} x$ **(50)** $2^x\log_2 x$

(51) $x^3\operatorname{arcsen} x+\cos x\log_3 x$ **(52)** $5^x\operatorname{tg} x+\mathrm{e}^{\,7}\operatorname{arctg} x$

(53) $x\cos x\log_4 x$ **(54)** $7^x\operatorname{arcsen} x\operatorname{tg} x$ **(55)** $\dfrac{1}{\log_2 x}$

(56) $\dfrac{1}{\operatorname{sen} x}$ **(57)** $\dfrac{\pi}{x+\operatorname{arcsen} x}$ **(58)** $\dfrac{3}{x\log x}$

(59) $\dfrac{x+2}{x^3-1}$ **(60)** $\dfrac{x+\log x}{\operatorname{arcsen} x}$ **(61)** $\dfrac{4^x+\sqrt{x}}{1+\sqrt{x}}$

(62) $\dfrac{\operatorname{arctg} x}{x+\operatorname{arcsen} x}$ **(63)** $\dfrac{x\operatorname{sen} x}{x^2+\operatorname{arcsen} x}$ **(64)** $\dfrac{1+\operatorname{tg} x}{1-\operatorname{tg} x}$

(65) $\dfrac{1+\cos x}{\cos x}$ **(66)** $\dfrac{\pi^2+x\log x}{1+\operatorname{sen} x}$ **(67)** $\dfrac{1+\sqrt{x}}{1-\sqrt{x}}$

(68) $\dfrac{x^8-3x^5+1}{x^3}$ **(69)** $\dfrac{x^3-2x+1}{\sqrt[4]{x}}$ **(70)** $\operatorname{sen} 2x$

(71) $\operatorname{tg} 3x$ **(72)** $\operatorname{cotg} 4x$ **(73)** $\mathrm{e}^{\,5x}$

(74) $\log|x|$ **(75)** $\operatorname{sen}^3 x$ **(76)** $\operatorname{tg}^2 x$

(77) $\sqrt{\operatorname{sen} x}$ **(78)** $\sqrt[4]{\log_2 x}$ **(79)** $\sqrt[3]{x^2-1}$

(80) $\sqrt{x^2+x+1}$ **(81)** $\sqrt[6]{\operatorname{arcsen} x}$ **(82)** $\dfrac{1}{\sqrt{\operatorname{tg} x}}$

(83) $\operatorname{arcsen}^4 x$ **(84)** $\log_5^3 x$ **(85)** 7^{x^2+x+1}

(86) $\pi^{\cos x}$ **(87)** $\mathrm{e}^{\sqrt{x}}$ **(88)** $3^{\operatorname{arcsen} x}$

(89) $5^{-\frac{1}{x^2}}$ **(90)** $\mathrm{e}^{\operatorname{tg} x}$ **(91)** $\sqrt{5^{2x}-5^x+1}$

(92) $\sqrt[4]{\log^2 x - \log x}$ **(93)** $\log_2(x^2-x)$ **(94)** $\log \operatorname{sen} x$

(95) $\log_3 \operatorname{arcsen} x$ **(96)** $\log(\mathrm{e}^{2x}-\mathrm{e}^x-6)$ **(97)** $\log_2 |\operatorname{sen} x|$

(98) $\log\log x$ **(99)** $\sqrt{\log_{\frac{1}{2}} \operatorname{arcsen} x}$ **(100)** $\log\left|\operatorname{tg}\dfrac{x}{2}\right|$

(101) $\operatorname{sen}\log_2 x$ **(102)** $\cos(x^2+x+1)$ **(103)** $\cos 2^x$

(104) $\operatorname{sen}(1-\sqrt{x})$ **(105)** $\operatorname{tg}(x^2+1)$ **(106)** $\operatorname{cotg}\sqrt{x}$

(107) $\operatorname{sen}\operatorname{arctg} x$ **(108)** $\operatorname{tg}^3 \operatorname{arcsen} x$ **(109)** $\sqrt{\operatorname{sen}\log x}$

(110) $\operatorname{arcsen}\log x$ **(111)** $\operatorname{arctg}(x^2+\pi)$ **(112)** $\operatorname{arcsen}\sqrt{x}$

(113) $\operatorname{arctg}^3 \log_2 x$ **(114)** $\operatorname{arctg}\operatorname{sen} x$ **(115)** $\arccos 4^{-\log x}$

(116) $\operatorname{arctg}\dfrac{1+x}{1-x}$ **(117)** $\operatorname{arcsen}\sqrt{1-x^2}$ **(118)** $3^{\sqrt{\operatorname{arcsen}\frac{1}{x}}}$

(119) $\operatorname{sen}\sqrt[3]{1+\log x}$ **(120)** $\log(1-\sqrt{1-x^2})$ **(121)** $\cosh\log x$

(122) $\operatorname{tgh}\sqrt{x}$ **(123)** $\sqrt[4]{\operatorname{tgh} x}$ **(124)** $\operatorname{settsenh}(x^2+x-\pi)$

(125) $\operatorname{setttgh}^2 \log x$ **(126)** $\operatorname{settcosh}\mathrm{e}^x$ **(127)** $(\arccos\log x)\cdot 3^{\operatorname{sen} x}$

(128) $\operatorname{arcsen}\dfrac{1-x}{1+x}$ **(129)** $(\cos\pi^x)\cdot\operatorname{arcsen}\log x$

(130) $\operatorname{arcsen}\dfrac{1}{\sqrt{1+x^2}}$ **(131)** $\log\log\log x$ **(132)** $\dfrac{2}{\sqrt[5]{\operatorname{arctg} 3^{\operatorname{sen} x}}}$

(133) $\operatorname{settsenh}^2 4^{\log x}$ **(134)** $\log_x(x^2-1)$ **(135)** $\log_x \operatorname{sen} x$

(136) $\log_{1+\sqrt{x}}(1+2^x)$ **(137)** $\log_{\operatorname{arctg} x}(x^2+\pi)$ **(138)** $\operatorname{arctg}\operatorname{tgh} x$

(139) $(\sqrt{x})^{\sqrt{x}}$ **(140)** $(\operatorname{arcsen} x)^x$ **(141)** $(1-\operatorname{sen} x)^x$

(142) $(1+\sqrt{x})^{\operatorname{sen} x}$ **(143)** $(2-x^4)^{x^2}$ **(144)** $(\cos x)^{\operatorname{sen} x}$

(145) $x^{\arccos x}$ **(146)** $(\operatorname{arctg} x)^x$

(147) Determinare quante soluzioni ha l'equazione

$$6\operatorname{sen} x - 6x + x^3 = 0\,.$$

(Si osservi che $x = 0$ è soluzione dell'equazione).

(148) Determinare il numero delle soluzioni reali dell'equazione:

$$3x^2 + 3\cos^2 x - x^4 = 1$$

(149) Determinare il numero di soluzioni dell'equazione

$$x + \log(1+x) - x^2 \log\Big(1+\frac{1}{x}\Big) = 1\,, \qquad x > 0 \quad .$$

(Sugg.: si ricordi che $\log(1+t) < t$ se $t > 0$)

(150) Determinare il numero di soluzioni dell'equazione

$$\left|\frac{x}{x^2-1}\right| + x = 2 \qquad x \in \mathbb{R}\,.$$

(151) Trovare il massimo e minimo della funzione

$$\frac{|x^2-1| - |x|}{x+2}$$

nell'intervallo [-1,2].

Studiare i grafici delle funzioni seguenti:

(152) $\sqrt[3]{x^2(x-1)}$ **(153)** $\mathrm{e}^{\frac{x-1}{x+1}}$ **(154)** $x|\log x|$

(155) $x^2 \operatorname{sen} x$ **(156)** $(x-1)^2(x-3)^2$ **(157)** $\dfrac{1-x}{1+x}$

(158) $\dfrac{1}{1+x^4}$ **(159)** $\dfrac{x^2-1}{x^2}$ **(160)** $\dfrac{2-3x-2x^2}{x}$

(161) $\dfrac{x^2}{x^2-x-2}$ **(162)** $\dfrac{x^3(x^2+2)}{x^4+x^2+1}$ **(163)** $\dfrac{x^4+2x^3}{x^3-1}$

(164) $|x^2-2x|$ **(165)** $|x|+|x^2-x|$ **(166)** $x\sqrt[3]{x^2}$

(167) $x+\sqrt{x+2}$ **(168)** $\sqrt[3]{x(x^2-1)}$ **(169)** $\sqrt{\dfrac{x^3-1}{x+2}}$

(170) $\dfrac{\sqrt[3]{x+1}+\sqrt[3]{x-1}}{\sqrt[3]{x+1}-\sqrt[3]{x-1}}$ **(171)** $x+\sqrt{x^2-1}$ **(172)** $\sqrt{|x|-|x-1|}$

(173) $x\,\mathrm{e}^{\frac{1}{x}}$ **(174)** $x\dfrac{1-2^x}{1+2^x}$ **(175)** $\dfrac{1}{(2^{\frac{1}{x}}-2)^2}$

(176) $x\log x$ **(177)** $\dfrac{1}{\log x}$ **(178)** $\log\log x$

(179) $\log \operatorname{arctg} x$ **(180)** $\dfrac{1}{x}+\log x$ **(181)** $\dfrac{1}{x\log x}$

(182) $x+\cos x$ **(183)** $\operatorname{arcsen}\dfrac{x+1}{x}$ **(184)** $x^{\frac{1}{x}}$

(185) $\dfrac{x^2}{x-1}$ **(186)** $\left|\dfrac{x}{1-x^2}\right|$ **(187)** $\dfrac{[x]}{x}$

(188) $\dfrac{[x]}{1-[x]}$ **(189)** e^{x^2} **(190)** $\mathrm{e}^{-\frac{1}{x^3}}$

(191) $\mathrm{e}^{-\mathrm{e}^{-x}}$ **(192)** $\mathrm{e}^{\frac{x}{1-x}}$ **(193)** $x\,\mathrm{e}^{\frac{x}{1-|x|}}$

(194) $\dfrac{1}{2^{\frac{1}{x}}+2^{\frac{1}{1-x}}}$ **(195)** $\log \operatorname{arcsen} x$ **(196)** $\dfrac{x}{\log x}$

(197) $\log|\operatorname{arctg} x|$ **(198)** $2x+\cos x$ **(199)** $x^2+\cos x$

(200) $\operatorname{sen}^2 x$ **(201)** $\operatorname{sen}|x|$ **(202)** $\operatorname{cotg}|x|$

(203) $2^{\cos x}$ **(204)** $x^2\,\mathrm{e}^{-\cos x}$ **(205)** $\log|\cos x|$

(206) $\operatorname{arctg} x-\dfrac{x}{1+x^2}$ **(207)** $\dfrac{\operatorname{arcsen} x}{x}$ **(208)** $\operatorname{arcsen}\log x$

(209) $\operatorname{arcsen}|x|$ **(210)** $\arccos(\cos\sqrt[3]{x})$ **(211)** $\dfrac{\operatorname{tg} x}{x}$

(212) $\dfrac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x}$ **(213)** $\operatorname{arctg}\dfrac{x^2}{|x-1|}$

Calcolare i seguenti limiti:

(214) $\displaystyle\lim_{x\to 0}\frac{1}{x}\left[\sqrt[3]{\frac{1-\sqrt{1-x}}{\sqrt{1+x}-1}}-1\right]$

(215) $\displaystyle\lim_{x\to 0}\frac{\log^2(1+x)-\log^2(1+\operatorname{sen}x)}{x(x-\operatorname{sen}x)}$

(216) $\displaystyle\lim_{x\to 0}\frac{\operatorname{arctg}^2\left[2\operatorname{sen}x-\log(1+x)\right]}{(2^x-1)\left(\sqrt{1+\operatorname{tg}x}-1\right)}$

(217) $\displaystyle\lim_{x\to 0}\left(\frac{1+\operatorname{sen}^2 x}{1-x}\right)^{\frac{1}{\operatorname{tg}x}}$

(218) $\displaystyle\lim_{x\to 0}\frac{[\log(1+x\operatorname{sen}x)]\cdot\operatorname{arctg}^2(\mathrm{e}^{\,x}-1)}{\sqrt[4]{1+3x^4}-1}$

(219) $\displaystyle\lim_{x\to 0^+}\frac{1-\cos x-\operatorname{sen}2x}{\pi^2-9\operatorname{arctg}^2\left(\frac{\sqrt{3}}{1+x}\right)}$

(220) $\displaystyle\lim_{x\to 0}\frac{\left(1+\operatorname{sen}x+\operatorname{sen}^2 x\right)^{\frac{1}{x}}-\left(1+\operatorname{sen}x\right)^{\frac{1}{x}}}{x}$

(221) $\displaystyle\lim_{x\to+\infty}\frac{x^4\operatorname{sen}^2(\pi-2\operatorname{arctg}x)}{3+x^2}$

(222) $\displaystyle\lim_{x\to 0}\frac{4^{\arccos^2\frac{1}{1+x^2}}-1}{\log_4(1+x)}$

(223) $\displaystyle\lim_{x\to 0}\frac{4\operatorname{sen}x-2\operatorname{tg}^3\sqrt[3]{3x}}{\cos x+\operatorname{sen}x-1}$

(224) $\displaystyle\lim_{x\to 0^+}\left[\frac{\pi}{2}+\operatorname{tg}x-\operatorname{arctg}\frac{1}{x}\right]^{\frac{1}{\log x}}$

(225) $\displaystyle\lim_{x\to 0^+}\frac{1}{x}\left[\left(-\frac{\log(1-x)}{\log(1+x)}\right)^{\alpha}-1\right]\quad\text{con}\quad\alpha>0$

Calcolare i seguenti limiti usando la formula di Taylor:

(226) $\displaystyle\lim_{x\to 0^+}\frac{\mathrm{e}^{\,x\cos x}-\log^2(1+\sqrt{x})-1}{\sqrt{\operatorname{sen}x-x\cos x}}$

(227) $\displaystyle\lim_{x\to 0}\frac{(1+\operatorname{sen}^2 x)^{\frac{1}{x}}-\mathrm{e}^{\,\operatorname{sen}x}}{x^3}$

(228) $\displaystyle\lim_{x\to 0}\frac{12\log\left(1+\operatorname{sen}\sqrt[3]{x}-\sqrt[3]{x}\right)+2x-\sqrt[3]{x^2}\left(\sqrt[10]{1+x}-1\right)}{\log^2\left(1+\sqrt{x}\right)+x\sqrt{x}-\operatorname{sen}x}$

(229) $\displaystyle\lim_{x\to 0} \frac{\operatorname{arctg}^2\big(e^x-1\big)-x^3-\log(1+x\operatorname{sen}x)}{\sqrt{1+3x^4}-1}$

(230) $\displaystyle\lim_{x\to 0} \frac{24\operatorname{arcsen}(1-\cos\sqrt{x})-12\log(1+x)-5x^2}{(1+\operatorname{sen}^2 x)^{\operatorname{tg}x}-1}$

(231) $\displaystyle\lim_{x\to 0} \frac{\big(1+x\big)^{\frac{2+x}{2x}}-e}{\log(1+x)+\operatorname{sen}^2 x-x}$

(232) $\displaystyle\lim_{x\to 0} \frac{e^{\operatorname{sen}x}+\cos x-2\Big(1+\operatorname{sen}\frac{x}{2}\Big)}{3\operatorname{tg}x-\operatorname{sen}3x}$

(233) $\displaystyle\lim_{x\to+\infty} x^2\log\left(1+\frac{1}{\sqrt[3]{x^2}}\right)\big[\operatorname{arctg}\sqrt[3]{x+1}-\operatorname{arctg}\sqrt[3]{x-1}\big]$

(234) $\displaystyle\lim_{x\to+\infty} \frac{(x^2+2)^2\log x+2x^3-x^4\log(x+2)}{x^2\log(1+x\operatorname{arctg}x)}$

(235) $\displaystyle\lim_{x\to 0} \frac{\arccos\log(\mathrm{e}-x^4)}{\pi-2\arccos(1+\mathrm{e}^{\,x^2}-2\cos x)}$

(236) $\displaystyle\lim_{x\to 0} \frac{\pi-2\operatorname{arcsen}[\log(\mathrm{e}-x^4)]}{\pi-\arccos[x\operatorname{sen}x-\log(1+x^2)-1]}$

(237) $\displaystyle\lim_{x\to 0} \frac{\displaystyle\int_0^x \frac{\operatorname{tg}t}{t}\,dt-x}{\log(1+\operatorname{tg}^2 x)^{\operatorname{cotg}x}-x}$

(238) $\displaystyle\lim_{x\to 0^+} \frac{(1+x^3)^{1/x}-\cos x}{\operatorname{sen}^2\sqrt{x}-\log^2(1+\sqrt{x})-x\sqrt{x}}$

(239) $\displaystyle\lim_{x\to+\infty}\Big(\frac{2\operatorname{arctg}x+\operatorname{tg}\frac{1}{x}}{\pi}\Big)^x$

(240) $\displaystyle\lim_{x\to 0} \frac{x-\displaystyle\int_0^x \frac{\operatorname{sen}t}{t}\,dt}{x\sqrt[5]{1+5x}-x^2-\operatorname{sen}x}$

(241) $\displaystyle\lim_{x\to+\infty} \frac{\operatorname{arctg}^2(x^2+x+1)-\operatorname{arctg}^2 x^2}{\operatorname{sen}\left(\frac{1}{x}+\frac{1}{x^3}\right)-\operatorname{sen}\left(\frac{1}{x}\right)}$

Determinare l'ordine di infinitesimo per $x\to 0$ e la parte principale di:

(242) $\log(1+\operatorname{sen}x)-\log(1+x)+x-\operatorname{sen}x$

(243) $\operatorname{sen}3x-x\cos 3x-2x$

(244) $(1+\operatorname{tg}^2 x)^{\frac{1}{x}}-e^{\operatorname{tg}x}$

(245) $\left(\dfrac{1-\sqrt{1-x}}{-1+\sqrt{1+x}}\right)^{\alpha}-1, \quad \alpha\in\mathbb{R}$

(246) $(1-x^3)^{\frac{1}{2x}}-\cos x$

(247) $\mathrm{e}^{3x}-\cos x-9x^2-\operatorname{sen}3x$

(248) $3\operatorname{arctg}x-2x\cos x-x$

(249) $\dfrac{1}{x^2}\log\left(1-\operatorname{tg}\left(x+\dfrac{\pi}{2}\right)\right)$

(250) $\log_2\left(3+\dfrac{1}{x^4}+2^{1/x}\right)$

(251) $\dfrac{\pi-2\operatorname{arctg}\frac{1}{x^2}}{\operatorname{sen}4x}$

(252) $\mathrm{e}^{x}-\mathrm{e}^{\operatorname{sen}x}$

(253) $\left(\mathrm{e}^{x}-\mathrm{e}^{\sqrt{1+2x}-1}\right)^2$

(254) $\tan^2 x-\operatorname{sen}^2 x\operatorname{arcsen}^2 x$

Calcolare i seguenti integrali immediati:

(255) $\displaystyle\int\frac{dx}{(2+3x)^4}$

(256) $\displaystyle\int\frac{dx}{\sqrt[5]{3x+4}}$

(257) $\displaystyle\int\operatorname{cotg}x\,dx$

(258) $\displaystyle\int x\,\mathrm{e}^{-x^2}\,dx$

(259) $\displaystyle\int\frac{dx}{\operatorname{arctg}^2 3x\cdot(1+9x^2)}$

(260) $\displaystyle\int x\sqrt{1-x^2}\,dx$

(261) $\displaystyle\int\frac{dx}{\operatorname{sen}x\cos x}$

(262) $\displaystyle\int\operatorname{sen}x\cos x\,dx$

(263) $\displaystyle\int\operatorname{sen}^4 x\cos x\,dx$

(264) $\displaystyle\int\frac{\log x}{x}\,dx$

(265) $\displaystyle\int\frac{dx}{\sqrt{\mathrm{e}^{2x}-4}}$

(266) $\displaystyle\int\frac{dx}{1-\operatorname{sen}x}$

(267) $\displaystyle\int\frac{\operatorname{sen}^3 x}{\cos^5 x}\,dx$

(268) $\displaystyle\int\frac{2^{\operatorname{tg}x}}{1+\cos 2x}\,dx$

(269) $\displaystyle\int\frac{x^2}{\sqrt{1-x^6}}\,dx$

(270) $\displaystyle\int\frac{\operatorname{sen}x\cos x}{\operatorname{sen}^4 x+\cos^4 x}dx$

(271) $\displaystyle\int\frac{dx}{\sqrt{x-x^2}}$

(272) $\displaystyle\int\frac{dx}{x\cos^2\log x}$

(273) $\displaystyle\int\frac{dx}{x\sqrt{1+x^2}}$

(274) $\displaystyle\int\frac{dx}{x^2+x+2}$

(275) $\displaystyle\int\frac{dx}{x^2-x-2}$

(276) $\displaystyle\int \frac{dx}{\sqrt{x^2-5x+6}}$ **(277)** $\displaystyle\int \frac{dx}{\sqrt{1-x-x^2}}$ **(278)** $\displaystyle\int \frac{dx}{1+\cos^2 x}\,dx$

Calcolare i seguenti integrali per decomposizione in somma:

(279) $\displaystyle\int \frac{3x+1}{2x-3}\,dx$ **(280)** $\displaystyle\int \frac{3x+2}{4+5x^2}\,dx$ **(281)** $\displaystyle\int \frac{dx}{x(4+x^2)}$

(282) $\displaystyle\int \sqrt{\frac{x-2}{x+2}}\,dx$ **(283)** $\displaystyle\int \frac{3x-2}{x^2-4x+3}\,dx$ **(284)** $\displaystyle\int \frac{x+2}{\sqrt{-x^2-x+2}}\,dx$

(285) $\displaystyle\int \frac{dx}{1+\mathrm{e}^{\,2x}}$ **(286)** $\displaystyle\int \frac{\sqrt{x^2+1}}{x}\,dx$ **(287)** $\displaystyle\int \operatorname{sen} x\cos 3x\,dx$

(288) $\displaystyle\int \cos 4x\cos 5x\,dx$ **(289)** $\displaystyle\int \operatorname{sen} x\cos 2x\operatorname{sen} 3x\cos 4x\,dx$

(290) $\displaystyle\int \operatorname{sen}^3 x\,dx$ **(291)** $\displaystyle\int \operatorname{sen}^4 x\cos^4 x\,dx$ **(292)** $\displaystyle\int \frac{dx}{\operatorname{sen}^3 x\cos x}$

(293) $\displaystyle\int \frac{dx}{\operatorname{sen}^2 x\cos^2 x}$ **(294)** $\displaystyle\int \frac{dx}{\operatorname{sen} x\cos^3 x}$ **(295)** $\displaystyle\int \operatorname{tg}^4 x\,dx$

Calcolare i seguenti integrali per parti:

(296) $\displaystyle\int x\operatorname{sen} x\,dx$ **(297)** $\displaystyle\int x\log x\,dx$ **(298)** $\displaystyle\int x\operatorname{sen} x\cos x\,dx$

(299) $\displaystyle\int \log x\,dx$ **(300)** $\displaystyle\int \operatorname{arctg} x\,dx$ **(301)** $\displaystyle\int \frac{x\operatorname{arcsen} x}{\sqrt{1-x^2}}\,dx$

(302) $\displaystyle\int \sqrt{1-x^2}\,dx$ **(303)** $\displaystyle\int \sqrt{x^2+x+3}\,dx$ **(304)** $\displaystyle\int x^2\,\mathrm{e}^{\,x}\cos x\,dx$

(305) $\displaystyle\int \mathrm{e}^{\,\operatorname{arcsen} x}\,dx$ **(306)** $\displaystyle\int x\operatorname{sen}^2 x\,dx$ **(307)** $\displaystyle\int x^2\,\mathrm{e}^{\,2x}\,dx$

(308) $\displaystyle\int x^3\cos x\,dx$ **(309)** $\displaystyle\int \log^2 x\,dx$ **(310)** $\displaystyle\int \operatorname{arcsen}^2 dx$

(311) $\displaystyle\int \frac{\operatorname{arctg} x}{x^2}\,dx$ **(312)** $\displaystyle\int \frac{dx}{(1+x^2)^3}$

Calcolare i seguenti integrali di funzioni razionali:

(313) $\displaystyle\int \frac{dx}{x^2-4}$ **(314)** $\displaystyle\int \frac{x^4+1}{x^3-x^2+x-1}\,dx$ **(315)** $\displaystyle\int \frac{x}{1+x^3}\,dx$

(316) $\displaystyle\int \frac{x}{1+x^3}\,dx$ **(317)** $\displaystyle\int \frac{x^2}{(x^2-1)(x+2)}\,dx$ **(318)** $\displaystyle\int \frac{dx}{1+x^4}$

(319) $\displaystyle\int \frac{2x^2+3x+4}{1-x^6}\,dx$ **(320)** $\displaystyle\int \frac{x^3+5x+2}{x^4(2x^2+3x+2)}\,dx$

(321) $\displaystyle\int \frac{2x^2+x+1}{x^2(x^2+x+1)}\,dx$ **(322)** $\displaystyle\int \frac{x^4}{(x^2-1)^3}\,dx$

(323) $\displaystyle\int \frac{dx}{x^3-7x+6}$ **(324)** $\displaystyle\int \frac{x}{x^4-x^2-2}\,dx$

(325) $\displaystyle\int \frac{dx}{(x^2+1)(x^2+2)}$ **(326)** $\displaystyle\int \frac{dx}{(x-1)^2(x-2)}$

(327) $\displaystyle\int \frac{dx}{1-x^4}$ **(328)** $\displaystyle\int \frac{dx}{x^4-7x^2-12}$ **(329)** $\displaystyle\int \frac{x^2}{x^4+x^2-2}\,dx$

(330) $\displaystyle\int \frac{2x^3+3}{x^5-9x}\,dx$

Calcolare i seguenti integrali per sostituzione:

(331) $\displaystyle\int x^3\,\mathrm{e}^{\,-x^2}\,dx$ **(332)** $\displaystyle\int \frac{x^3}{\sqrt{1-x^2}}\,dx$ **(333)** $\displaystyle\int \frac{\mathrm{e}^{\,3x}+\mathrm{e}^{\,x}}{1+\mathrm{e}^{\,2x}}\,dx$

(334) $\displaystyle\int \frac{\operatorname{sen} x+\cos x}{2\operatorname{sen} x-3\cos x}dx$ **(335)** $\displaystyle\int \frac{\sqrt{x}}{x-1}\,dx$ **(336)** $\displaystyle\int \frac{dx}{x^2\sqrt{x+1}}$

(337) $\displaystyle\int \frac{1+\sqrt[6]{x}}{\sqrt[3]{x}(1+\sqrt[4]{x})}\,dx$ **(338)** $\displaystyle\int \frac{dx}{2x^2+2x+3}$ **(339)** $\displaystyle\int \frac{\sqrt{x^2-1}}{x}\,dx$

(340) $\displaystyle\int \frac{dx}{(x+1)\sqrt{x^2+x+1}}$ **(341)** $\displaystyle\int \frac{1+\sqrt{x}}{1+\sqrt[3]{x}}\,dx$ **(342)** $\displaystyle\int \frac{1+\cos^2 x}{(1+\cos x)\operatorname{sen} x}dx$

(343) $\displaystyle\int \frac{\cos^2 x}{\operatorname{sen}^4 x+\cos^2 x}\,dx$

Calcolare i seguenti integrali:

(344) $\displaystyle\int \mathrm{e}^{\,-x}\sqrt{1+\mathrm{e}^{\,x}}\,dx$ **(345)** $\displaystyle\int \mathrm{e}^{\,x}\cos^3 x\,dx$

Calcolare i seguenti integrali definiti:

(346) $\displaystyle\int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \frac{x^2}{x^4-1}\,dx$ **(347)** $\displaystyle\int_0^{\pi} \operatorname{sen} 3x \cos x\,dx$ **(348)** $\displaystyle\int_0^1 \sqrt{1-x^2}\,dx$

(349) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{(1-\cos x)\operatorname{tg}\frac{x}{2}}{1+\cos^2 x}\,dx$ **(350)** $\displaystyle\int_1^{\sqrt{2}} \frac{dx}{1+\sqrt{x^2-1}}$

(351) $\displaystyle\int_0^{\pi} \operatorname{sen}^4 x\,dx$ **(352)** $\displaystyle\int_0^1 x^2(1-x^2)^{\frac{3}{2}}\,dx$ **(353)** $\displaystyle\int_0^{\frac{\pi}{4}} \operatorname{tg}^4 x\,dx$

(354) $\displaystyle\int_0^{\frac{\pi}{2}} \operatorname{sen}^4 x \cos^3 x dx$ **(355)** $\displaystyle\int_{-1}^1 \frac{\sqrt{1-x^2}}{2-x}\,dx$ **(356)** $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\operatorname{sen} x-\cos x}{1+\operatorname{sen} x\cos x}dx$

(357) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\operatorname{sen}^2 x}{2\operatorname{sen}^2 x+3\cos^2 x}\,dx$ **(358)** $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{dx}{\operatorname{sen}^5 x}$

(359) $\displaystyle\int_0^{+\infty} \frac{dx}{1+x^2}$ **(360)** $\displaystyle\int_0^{+\infty} \frac{dx}{1+x^3}$ **(361)** $\displaystyle\int_0^1 \log x\,dx$

(362) $\displaystyle\int_0^{+\infty} x^3\,\mathrm{e}^{-x^2}\,dx$ **(363)** $\displaystyle\int_0^{+\infty} \frac{dx}{\sqrt{\mathrm{e}^{2x}+\mathrm{e}^x+1}}$ **(364)** $\displaystyle\int_0^{+\infty} \frac{dx}{1+x^4}$

(365) $\displaystyle\int_0^{+\infty} \frac{dx}{(x^2+1)(x^2+4)}$ **(366)** $\displaystyle\int_1^{+\infty} \frac{dx}{x\sqrt{x^2-1}}$ **(367)** $\displaystyle\int_0^1 x^{\frac{3}{2}}\log x\,dx$

(368) $\displaystyle\int_0^{+\infty} \frac{dx}{\mathrm{e}^x+3\,\mathrm{e}^{-x}}$

(369) Dire per quali valori di $\alpha \in \mathbb{R}$

$$\int_0^2 \frac{d\,x}{\sqrt{x}|x-1|^\alpha}$$

è finito e calcolarne il valore per $\alpha = 1/2$.

(370) Calcolare

$$\int_0^1 16\sqrt{x}\operatorname{arcsen}\sqrt{x}\,dx\ .$$

(371) Calcolare

$$\int_1^{+\infty} \frac{(3+x^2)(\arctan^2 x-\arctan x)}{x^4}dx\ .$$

(372) Dire per quali $\alpha \in \mathbb{R}$ è finito l'integrale

$$\int_0^{+\infty} \frac{x}{(1+x^3)^\alpha}\,dx$$

e calcolarne il valore per $\alpha = 1$.

(373) Sia $\alpha \in \mathbb{R}$; dire per quali valori di α la funzione

$$\frac{\sqrt{x}(x-4)}{x^\alpha(1+x)^2}$$

è sommabile sull'intervallo $(0, +\infty)$; calcolare il valore dell'integrale per $\alpha = 1$.

Studiare la convergenza delle seguenti serie a termini positivi:

(374) $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{2^n}$

(375) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\mathrm{e}^{\,n}}$

(376) $\displaystyle\sum_{n=1}^{\infty} \operatorname{sen}^{\,2} \frac{1}{n}$

(377) $\displaystyle\sum_{n=1}^{\infty} \operatorname{sen} \frac{1}{n}$

(378) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n n!}{n^n}$

(379) $\displaystyle\sum_{n=1}^{\infty} \left(\sqrt[n]{n} - 1\right)^n$

(380) $\displaystyle\sum_{n=1}^{\infty} \frac{1 + 2\operatorname{sen}^{\,2} n}{n^2}$

(381) $\displaystyle\sum_{n=1}^{\infty} \left(1 - \cos\frac{1}{n}\right)$

(382) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(n+1)^3}$

(383) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n(2n+2)}$

(384) $\displaystyle\sum_{n=1}^{\infty} \frac{3 - 2\cos n}{2^n}$

(385) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+5}}$

(386) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\log n}$

(387) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{(2n)!}$

(388) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n^2 - n - 5}$

(389) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{3n^2 - n - 1}$

(390) $\displaystyle\sum_{n=1}^{\infty} \frac{3n-2}{n^3 - n^2 + 2}$

(391) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n + \sqrt{n}}$

(392) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(\log n)^n}$

(393) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(\log n)^{\log n}}$

(394) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(1+n)}$

(395) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{1+n^2}$

(396) $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{n^n}$

(397) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)}}$

(398) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(1+n^2)}}$

(399) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

(400) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt[n]{n}}$

(401) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n + 5\log^3 n}$

(402) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(\log n)^\alpha},\ \alpha > 0$

(403) $\displaystyle\sum_{n=1}^{\infty} \log\left(1 + \frac{n+1}{n^3+n+3}\right)$

(404) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \operatorname{sen}\frac{1}{\sqrt{n}}\right)$

(405) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\log\left(1 + \frac{1}{n}\right)}$

(406) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n^5}{3^n}$

(407) $\displaystyle\sum_{n=1}^{\infty} n^3\,\mathrm{e}^{\,-n}$

(408) $\displaystyle\sum_{n=1}^{\infty} 2^n n^3 \,\mathrm{e}^{\,-n}$ **(409)** $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n+2}{3n+1}\right)^n$ **(410)** $\displaystyle\sum_{n=3}^{\infty}\left(\frac{1}{\log\log n}\right)^{\log n}$

(411) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n\log^2(n^2+1)}$ **(412)** $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n(\log n)^\alpha},\ \alpha>0$

(413) $\displaystyle\sum_{n=3}^{\infty}\frac{1}{n\log n(\log\log n)^\alpha},\ \alpha>0$

Studiare la convergenza assoluta delle seguenti serie:

(414) $\displaystyle\sum_{n=1}^{\infty}\frac{2+(-1)^n n}{2^n}$ **(415)** $\displaystyle\sum_{n=1}^{\infty}\frac{\operatorname{sen} n}{n^2}$ **(416)** $\displaystyle\sum_{n=1}^{\infty}\frac{1+(-1)^n n}{n^4+1}$

(417) $\displaystyle\sum_{n=1}^{\infty}\frac{\operatorname{sen} nx}{n^2}$ **(418)** $\displaystyle\sum_{n=1}^{\infty}\frac{2^n x^n}{n^2+1},\ x\in\mathbb{R}$ **(419)** $\displaystyle\sum_{n=1}^{\infty} nx^n,\ x\in\mathbb{R}$

(420) $\displaystyle\sum_{n=1}^{\infty}\frac{(n+1)x^n}{n!},\ x\in\mathbb{R}$

Studiare la convergenza delle seguenti serie a segni alterni:

(421) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \operatorname{sen}\frac{1}{n}$ **(422)** $\displaystyle\sum_{n=1}^{\infty}(-1)^n \operatorname{arctg}\frac{1}{2n+1}$ **(423)** $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n+(-1)^n}}$

(424) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{n+1}{4^n}$ **(425)** $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{(n+1)^2}$ **(426)** $\displaystyle\sum_{n=2}^{\infty}\log\left(1+\frac{(-1)^n}{n}\right)$

(427) Studiare la serie:

$$\sum_{n=1}^{\infty} n\int_0^{\frac{1}{n}}\frac{\sin^\alpha(t\sqrt{t})}{t}\,dt$$

dove $\alpha>0$.

(428) Data la successione

$$a_1=\alpha>0,\qquad a_{n+1}=\frac{n}{1+2n}a_n$$

mostrare che
(i) la successione $\{a_n\}$ è decrescente e infinitesima ;
(ii) la serie $\displaystyle\sum_{n=1}^{\infty} a_n$ è convergente.

(429) Dire per quali valori di $\alpha \geq 0$ converge la serie

$$\sum_{n=1}^{\infty} n^2 \Big(\operatorname{sen} \frac{1}{n^\alpha} - \frac{1}{n^\alpha + 1}\Big) \, .$$

(430) Dire per quali valori di $\alpha \in \mathbb{R}$ converge la serie

$$\sum_{n=2}^{+\infty} \frac{\log^2 \left(1 + \dfrac{1}{n^\alpha}\right)}{\log n \, \log^2 \log n}$$

(431) Studiare la serie

$$\sum_{n=1}^{\infty} (\sqrt{n^2 + n} - n) \Big(n \operatorname{tg} \frac{1}{n} - 1\Big)$$

(432) Studiare la convergenza della serie

$$\sum_{n=1}^{+\infty} \frac{e^n n!}{n^n} \quad .$$

(Sugg.: si provi che la successione che genera la serie è monotòna)

(433) Studiare la convergenza della serie

$$\sum_{n=1}^{+\infty} n \left(\log\Big(1 + \frac{1}{n}\Big) - \sin \frac{1}{n}\right) \quad .$$

Integrare le seguenti equazioni differenziali lineari:

(434) $y' = \dfrac{xy + 1}{x^2}$ **(435)** $y' = y - x^2$ **(436)** $y' = \dfrac{1 - y}{x}$

(437) $y' = \dfrac{1}{x^2} - \dfrac{y}{x}$ **(438)** $y' + (1 + \operatorname{cotg} x)y + \dfrac{x}{\operatorname{sen} x}$

(439) $y' = y + \mathrm{e}^{\,x}$ **(440)** $y' = \dfrac{y}{x^2-1} + x$ **(441)** $y' = \dfrac{\mathrm{e}^{\,x} - y}{x}$

(442) $y' = y \operatorname{tg} x$ **(443)** $y' = \dfrac{x+y}{x}$ **(444)** $y' - \dfrac{2}{x}y = 1$

(445) $y' + y\cos x = \dfrac{1}{2}\operatorname{sen} 2x$ **(446)** $y' + y \operatorname{cotg} x = \dfrac{\cos^3 x}{\operatorname{sen}^2 x + 1}$

(447) $y' + 2y = \mathrm{e}^{\,x}$ **(448)** $y' + y \operatorname{cotg} x = x^3$ **(449)** $xy' - y = -\log x$

(450) $y' - \dfrac{xy}{x^2-1} = x$

Integrare le seguenti equazioni a variabili separabili:

(451) $y' = \dfrac{x}{y}$ **(452)** $y' = \dfrac{2}{xy}$ **(453)** $y' = \dfrac{x^2+1}{(x^2-1)xy}$

(454) $y' = \operatorname{sen} x\sqrt{y+1}$ **(455)** $y' = \mathrm{e}^{\,x-y}$ **(456)** $y' = \dfrac{1-y^2}{1-x^2}$

(457) $y' = x\,\mathrm{e}^{\,y}$ **(458)** $(1+x^2)y' + 1 + y^2 = 0$

(459) $y' = \sqrt{y+1} + \sqrt{y-1}$ **(460)** $y' = -\dfrac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

(461) $xy' = y(1+y^2)$ **(462)** $y' = \dfrac{y^2}{1+x^2}$ **(463)** $y' = x \operatorname{tg} y$

(464) $y' = \dfrac{\sqrt{1+y^2}}{x}$ **(465)** $(x^2-1)y' + (1-y^2) = 0$

(466) $y' \operatorname{sen} x \cos y + 1 = 0$ **(467)** $(1-x^2)y' = xy$ **(468)** $y' = \dfrac{1}{\operatorname{tg} x \operatorname{tg} y}$

(469) $x^2y' + y = 1$ **(470)** $y' = \dfrac{y^2-y+1}{x^2+x+1}$ **(471)** $y' + \dfrac{\operatorname{sen} x}{\operatorname{sen} y} = 0$

(472) $y' = \operatorname{sen} x \cos y$

Integrare le seguenti equazioni differenziali lineari:

(473) $y'' - y = 0$ **(474)** $y'' + y' = 0$ **(475)** $y'' - y' + 2y = 0$

(476) $y'' + 2y' + y = 0$ **(477)** $y'' - 2y' + 5y = 0$ **(478)** $y'' + y' + y = 0$

(479) $y'' + y' - 6 = 0$ **(480)** $y'' + \omega^2 y = x^2$ **(481)** $y''+y'+y=\operatorname{sen} 2x$

(482) $y'' - y = x\,\mathrm{e}^{-x}$ **(483)** $y'' - y' + y = \mathrm{e}^{x}$ **(484)** $y'' + y = \cos 2x$

(485) $y'' + \omega^2 y = \operatorname{sen} \omega x$ **(486)** $y'' - 2y = 3\,\mathrm{e}^{2x}$ **(487)** $y'' + y = \dfrac{1}{\operatorname{sen} x}$

Risolvere i seguenti problemi di Cauchy:

(488) $\begin{cases} y' = y - \mathrm{e}^{-x} \\ y(0) = 1 \end{cases}$ **(489)** $\begin{cases} y' = \mathrm{e}^{x+y} \\ y(1) = 0 \end{cases}$

(490) $\begin{cases} y'' - y' - 2y = 0 \\ y(0) = 0,\ y'(0) = 1 \end{cases}$ **(491)** $\begin{cases} y'' + y' + y = x \\ y(0) = -1,\ y'(0) = 1 \end{cases}$

(492) $\begin{cases} y'' - 2y' + y = \dfrac{\mathrm{e}^{x}}{x} \\ y(1) = 0,\ y'(1) = 0 \end{cases}$ **(493)** $\begin{cases} y'' + y = x^2 \\ y(0) = -2,\ y'(0) = 0 \end{cases}$